

The Government Spending Multiplier, Fiscal Stress and Risk*

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Abstract

According to a growing empirical literature, the government spending multiplier appears to be relatively small in times of fiscal stress. I employ a medium-scale DSGE model with leverage constrained banks and sovereign default risk to analyze how the presence of fiscal stress can affect the transmission of government spending shocks. I find that the role of fiscal stress for the size of the government spending multiplier is negligible when analyzing the linearized economy. When the model is solved using a third-order approximation to equilibrium dynamics, however, and the effects of risk on the transmission of a government spending shock are therefore accounted for, the presence of fiscal stress can lead to a substantial decrease of the government spending multiplier, which is in line with the empirical evidence. For plausible calibrations of the model, the cumulative multiplier can even become negative.

Keywords: government spending multiplier, fiscal stress , aggregate risk, financial frictions

JEL Classification: E32, E 44, E62, H30, H60

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1. Introduction

During the recent sovereign debt crisis in the euro area several European countries have experienced sharp increases of their debt-to-GDP ratios and their sovereign bonds yields. To strengthen the sustainability of their public finances, these countries started programs of fiscal consolidation which included steep government spending cuts. This raises the question: How does fiscal stress affect the size of the government spending multiplier? A number of empirical papers find that the government spending multiplier is smaller, when public finances are weak.¹ This paper propose a medium scale DSGE model that offers an explanation for this empirical link between the degree of fiscal stress and the size of the government spending multiplier.

A key feature of this model are endogenously leverage constrained banks as in [Gertler and Karadi \(2011\)](#), which, in addition to private capital assets, hold government bonds that are subject to default risk. As stressed by [van der Kwaak and van Wijnbergen \(2014\)](#), the exposure of banks in the euro area to domestic sovereign bonds is substantial. Thus, fluctuations in the degree of fiscal stress are likely to affect the conditions of the banks' asset side, and therefore their ability to supply loans to firms. Indeed, as highlighted by [Corsetti, Kuester, Meier, and Müller \(2013\)](#), sovereign risk spreads and private credit spreads have been highly correlated in those countries in the euro area, which experienced increasing degrees of fiscal stress. Introducing the link between fragile banks and risky government bonds into the model allows me to capture that variations in government spending, to the extent that they affect the degree of fiscal stress, can have an immediate impact on the credit supply, and therefore on investment in the economy. Fiscal stress is captured in the model by the probability of a sovereign default modeled in the form of a fiscal limit function as discussed by [Leeper and Walker \(2011\)](#) and used in similar variations by other authors.² This function maps the debt-to GDP ratio into the probability of a sovereign default. In this setting, I analyze government spending multipliers obtained by a first-order approximation, as well as a third-order approximation to equilibrium dynamics. using the non-linear moving average approach by [Lan and Meyer-Gohde \(2013\)](#). Employing a third-order approximation implies that the degree of aggregate risk affects the equilibrium dynamics, and hence the response of the economy to a government spending shock. As a consequence, the precautionary motive of agents is accounted for.

I find that the degree of fiscal stress does affect the government spending multiplier in my model. An increase in government spending increases both, aggregate output and public debt. Initially, the debt-to-GDP ratio, and therefore also the sovereign default probability, decrease due to the output stimulus triggered by the increase in government spending. After a few quarters, however, as the stimulus fades out and public debt remains high, the debt-to-GDP ratio and the sovereign default probability increase above pre-shock level. The increase in the default probability raises the interest rate on government bonds, and as prices of capital assets and bonds are determined jointly in equilibrium, they also raise the interest rate on loans that the bank extend to firms. Thus, the increase in fiscal stress contributes to a crowding out of investment, dampens the output stimulus, and lowers the government spending multiplier.

However, whether this effect is quantitatively important depends on the way the model is solved. In the linearized economy, the effect of this 'fiscal stress channel' on bond prices and on the output response to the shock is negligible. In contrast, when the model is solved with a

¹see, e.g. [Perotti \(1999\)](#), [Corsetti, Meier, and Müller \(2012\)](#) and [Ilzetzki, Mendoza, and Végh \(2013\)](#)

²Examples include: [Bi and Traum \(2012a\)](#), [Bi and Traum \(2012b\)](#), [Corsetti et al. \(2013\)](#), [van der Kwaak and van Wijnbergen \(2013\)](#), [van der Kwaak and van Wijnbergen \(2015\)](#), [Bi, Leeper, and Leith \(2014\)](#).

third-order approximation, and therefore the role of aggregate risk for the transmission of the shock is accounted for, the 'fiscal stress channel' can become sizable.

The financial accelerator embedded in the model, and the probability of a sovereign default, are key to generating a relevant effect of risk on the equilibrium dynamics. A positive government spending shock worsens the balance sheet position of risk averse banks. When the risk of future shocks, that have the potential to further weaken the balance sheet of banks, is accounted for, the banks reduce their exposure to the risky asset to a larger degree. The fall in credit supply and investment is stronger, and the output stimulus is smaller and more short-lived than in a linear world, where the degree of risk has no impact on the dynamics of the model. The smaller output stimulus leads to a more immediate and more pronounced increase in the debt-to-GDP ratio, and hence also the sovereign default probability. This in turn further aggravates the balance sheet position of banks, reducing the credit supply, and further decreasing the government spending multiplier. The fiscal stress channel is more important for the government spending multiplier, the higher the debt-to-GDP ratio, the higher the sensitivity of fiscal stress to the debt-to-GDP ratio, and the higher the leverage ratio of banks. When the debt-to-GDP ratio is calibrated to 1.3, roughly matching the debt-to-GDP ratio observed in Italy in the recent years, the cumulative multiplier even becomes negative. Hence, solving the model non-linearly opens the door for a quantitatively relevant role of fiscal stress, and helps to explain why in the presence of higher fiscal stress, the government spending multiplier can be smaller.

A number of empirical papers find that the government spending multiplier is smaller, when public finances are weak. Analyzing data from a panel of 19 OECD countries from 1965 to 1994, [Perotti \(1999\)](#) finds evidence of substantially smaller government spending multipliers for countries that are characterized by high public debt levels or deficits. This finding of smaller multipliers in the presence of higher debt is corroborated by [Corsetti et al. \(2012\)](#), [Guajardo, Leigh, and Pescatori \(2014\)](#), and [Ilzetzki et al. \(2013\)](#). The latter find surprisingly large, negative long run multipliers for countries with high public debt. [Klein \(2016\)](#) finds that high public debt levels reduce the government spending multiplier, regardless of whether the stock of private debt in the economy is high or low. Recently, [Born, Müller, and Pfeifer \(2015\)](#) and [Strobel \(2016\)](#) employ regime-switching SVARs to analyze the effects of government spending shocks, when risk premia on government bonds are high. [Born et al. \(2015\)](#) analyze a panel of 38 developing and developed countries and find that in periods of higher risk premia on government bonds, the government spending multiplier are higher than in periods with lower risk premia. However, they acknowledge the challenge of disentangling the effects of higher sovereign spreads and the effects of recessions on the multiplier in their sample.³ To circumvent this difficulty, [Strobel \(2016\)](#) focusses on the case of Italy, between 1993 and 2013, as in this sample the correlation between fiscal stress, measured by the risk spreads on Italian government bonds, and output growth is low. Here, cumulative government spending multipliers for a time horizon of three years are significantly lower in a regime with higher sovereign yield spreads. Thus overall, the literature tends to find smaller multipliers in the face of fragile public finances.

This paper is not alone in analyzing the relation between fiscal stress and the government spending multiplier. The paper closest to mine is [van der Kwaak and van Wijnbergen \(2015\)](#). As I do in this paper, they employ the model framework by [Gertler and Karadi \(2011\)](#) augmented by risky long-term government bonds to investigate the link between the government spending multiplier and sovereign default risk. They find that due to the leverage constraint on banks,

³In contrast to fiscal stress, recessions are associated with higher government spending multipliers. (See, e.g. [Christiano, Eichenbaum, and Rebelo \(2011\)](#), [Auerbach and Gorodnichenko \(2012\)](#))

deficit financed government spending crowds out loans to private firms, which decreases the government spending multiplier. Furthermore, a longer duration of government bonds and the presence of default risk decreases the effectiveness of government spending. While my paper confirms their main findings, it differs from theirs along two dimensions: First, while in their model fiscal stress is determined by the level of public debt, in mine the fiscal stress indicator is a function of the debt-to-GDP ratio. This changes the dynamic response of the fiscal stress indicator to the government spending shocks, as the output stimulus after the shock initially decreases the debt-to-GDP ratio. As a consequence, in my linearized model, the link between fiscal stress and the government spending multiplier is drastically weaker than it is in [van der Kwaak and van Wijnbergen \(2015\)](#). Secondly, I additionally demonstrate the effect of risk on the size of the multiplier, which in my framework allows me to generate the negative relation between the degree of fiscal stress and the multiplier found in the empirical literature.

Another recent theoretical paper on government spending and the presence of fiscal stress is [Corsetti et al. \(2013\)](#). They argue that in the presence of fiscal stress, fiscal retrenchment, insofar as it stabilizes public finances, may also enhance the stability of the macroeconomy. In the extreme case this may even imply a negative government spending multiplier. The possibility of an expansionary fiscal contraction was originally proposed by [Giavazzi and Pagano \(1990\)](#). [Bertola and Drazen \(1993\)](#) build a model in which a fiscal retrenchment improves the expectations of households on the future growth path and increases consumption. [Alesina and Perotti \(1997\)](#) suggest that lower risk premia provide a channel through which fiscal contraction may stimulate investment and economic growth.⁴

A number of panel data studies investigate the drivers of sovereign yield spreads. Examples include [Alesina, de Broeck, Prati, and Tabellini \(1992\)](#), [Bernoth, von Hagen, and Schuknecht \(2003\)](#), [Manganelli and Wolswijk \(2009\)](#), [di Cesare, Grande, Manna, and Taboga \(2012\)](#), [Beirne and Fratzscher \(2013\)](#) and others. Generally, these studies find that higher public debt and lower growth raise risk premia in the same period. [Beirne and Fratzscher \(2013\)](#) suggest that there was a contagion effect spilling over from Greece. Among others, they argue that there has been a "fundamental contagion" which caused the markets to become more aware of the economic fundamentals in periphery countries. As only the periphery countries with high public debt and low GDP growth were affected by these spells, the results of this literature point towards an increase of the sensitivity of default risk to economic fundamentals in crisis time. In my framework, I model the relationship between fundamentals and the probability of a sovereign default in the form of a fiscal limit function à la [Leeper and Walker \(2011\)](#). I use a logistic distribution function, which increases with the debt-to-GDP ratio, and calibrate it using parameter values from the structural estimation on Italian data by [Bi and Traum \(2012a\)](#). The shape of this function allows for capturing the empirically observed non-linear relationship between fundamentals and sovereign yield spreads.

The role of aggregate risk for equilibrium dynamics in DSGE models is investigated by a strand of the literature, which was started by [Bloom \(2009\)](#).⁵ The focus of this literature lies on the effects of stochastic exogenous volatility. Capturing the effects of volatility shocks requires a third-order approximation to the equilibrium dynamics. As the focus of the paper at hand is the analysis of a government spending shock, I abstract from volatility shocks (often called "risk

⁴Other examples of papers that discuss explain the possibility of expansionary fiscal contraction are: [Blanchard \(1990\)](#) and [Sutherland \(1997\)](#). For empirical evidence in support of this hypothesis, see: e.g., [Giavazzi and Pagano \(1990\)](#), [Alesina and Ardagna \(2010\)](#) and [Bergman and Hutchison \(2010\)](#). Studies who cast doubt on the empirical evidence for the expansionary contraction hypothesis are [Perotti \(2011\)](#) and [Guajardo et al. \(2014\)](#).

⁵see also, e.g., [Fernandez-Villaverde, Guerrón-Quintana, Rubio-Ramirez, and Uribe \(2011\)](#), [Basu and Bundick \(2015\)](#) [Bachmann and Bayer \(2013\)](#)

shocks” or “uncertainty shocks”) and hold the degree of risk constant. Typically, an impulse response function to a level shock is an object for which a first-order approximation is regarded to be sufficient for its analysis. In my framework however, the financial accelerator mechanism together with the presence of fiscal stress generate sufficient amplification, so that the impact of aggregate risk for the transmission of the government spending shock is strong enough to create a non-negligible difference between multipliers generated with a first-order approximation and a third-order approximation. In turn, taking into account risk and the precautionary behavior of agents, allows me to generate the negative relationship between the degree of fiscal stress, and the size of the government spending multiplier, that is found in the empirical literature.

The remainder of the paper is structured as follows: Section two gives an overview of the model. Section three discusses the calibration and the solution method. The fourth section provides an anatomy of the dynamic consequences of government spending shocks, and an analysis of the dynamic effects of government spending shocks for the model with fiscal stress . It compares linear impulse response functions with their non-linear counterpart, and compares the dynamics of a model with fiscal stress to a model, in which the effects of fiscal stress for the multiplier are shut off. Section five highlights conditions under which the fiscal stress channel is particularly important for the government spending multiplier, and section six concludes.

2. The model

The environment

The model builds on the framework used by [Gertler and Karadi \(2011\)](#), and adds an extra twist to make it suitable for the analysis of the link between fiscal stress and the government spending multiplier. In particular, in addition to capital assets, banks hold government bonds as a second asset on their balance sheets, and the default probability of government bonds is tied to the debt-to-GDP ratio.

Time is discrete, and one period in the model represents one quarter. The model features households, banks, intermediate good producers, capital good producers, retailers, a fiscal and a monetary authority. Figure (1) provides an illustration of the model structure. Households consume, supply labor, and save in the form of bank deposits. The firm sector consists of three types of firms. Intermediate good producers employ labor and capital to produce their goods. Each period, after producing their output, they sell their used capital stock to the capital goods producers. The latter repair it, and invest in new capital. At the end of the period they re-sell the capital to the intermediate good producers, which use it for production in the next period. The intermediate good producers finance their purchases of capital with loans from the banks. Intermediate goods are purchased by retailers, which repackage them, and sell them with a markup as final goods to households, the capital producers, and to the government. Banks hold loans and government bonds on the asset side of their balance sheets. On the liability side are deposits and the banks net worth. The government consumes final goods, collects taxes, and issues government bonds, which are subject to default risk. Monetary policy takes the form of a Taylor rule.

The model includes habit formation in consumption, convex investment adjustment costs, variable capital utilization, sticky prices, and price indexation to enhance the empirical plausibility of the model dynamics, and to facilitate the comparability of my results with the results by other authors which have used this framework.

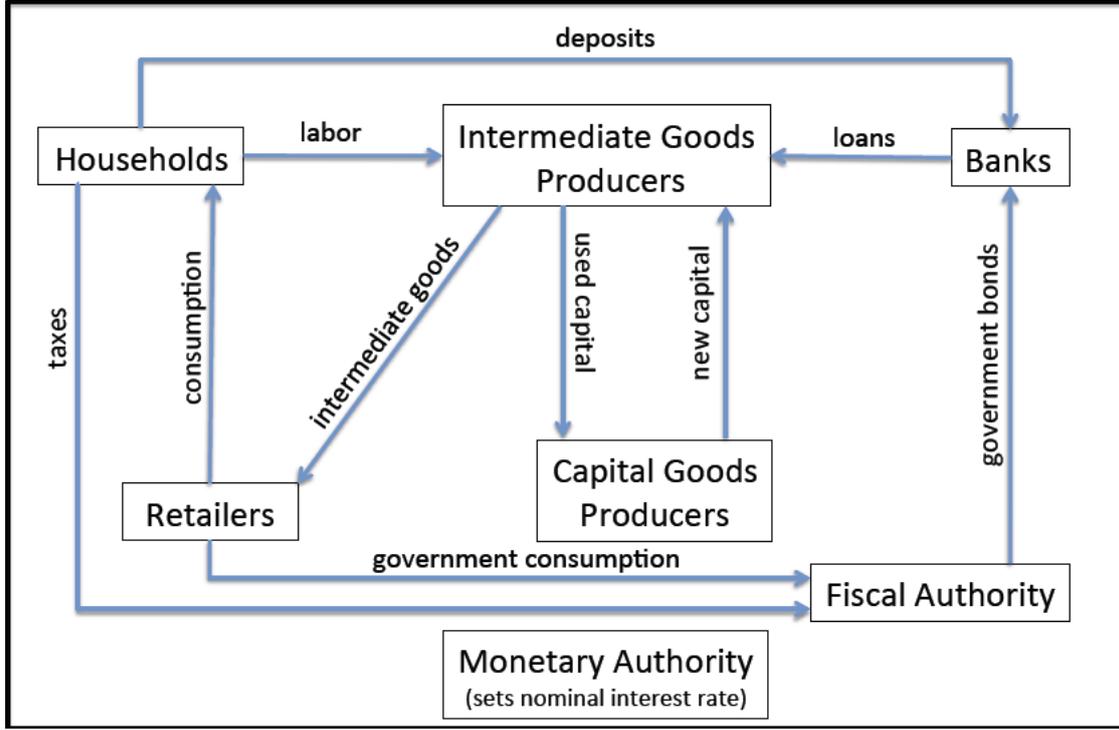


Figure 1: Overview of the model

Households

There is a continuum of households with a unit mass. As in [Gertler and Karadi \(2011\)](#) a constant fraction f of each household's members works as banker, whereas the other fraction $(1 - f)$ consists of workers who supply labor to the intermediate good producers. While workers receive their wage income every period, bankers reinvest their gains in asset holdings of the bank over several periods, and contribute to the households income only when exiting the banking sector, bringing home the accumulated profits. To ensure that both fractions of the household face the same consumption stream, perfect consumption insurance within the household is assumed. Households' expected lifetime utility is as follows

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t - hC_{t-1}) - \frac{\chi}{1+\phi} L_t^{1+\phi} \right],$$

where C_t is consumption and L_t is labor that the workers supply to intermediate good producers. β is the discount factor, h is the parameter of the habit formation, ϕ is the inverse of the Frisch elasticity, and χ scales the weight of the disutility from labor in the preferences. Households can save via a one period bank deposit, which earns the riskless interest rate, R_t . The income stream of the household is thus composed of the wage income $W_t L_t$, banker's profits Y_t^b , firm profits, Y_t^f net the payment of lump sum taxes T_t . It uses this income to purchase consumption goods or to renew its deposits. The budget constraint thus reads

$$C_t + D_t = W_t L_t + R_{t-1} D_{t-1} + Y_t^b + Y_t^f - T_t.$$

Maximizing life-time utility with respect to consumption, labor and deposit holdings subject to

the sequence of budget constraints yields the first order conditions of the household

$$W_t = \frac{\chi L_t^\phi}{U_{c,t}}, \quad (1)$$

$$U_{c,t} = (C_t - hC_{t-1})^{-1} - \beta h E_t (C_{t+1} - hC_t)^{-1}, \quad (2)$$

$$1 = E_t \beta \Lambda_{t,t+1} R_t, \quad (3)$$

with

$$\Lambda_{t,t+1} = \frac{U_{c,t+1}}{U_{c,t}}. \quad (4)$$

Firm sectors

The model contains three types of firms. Intermediate goods are produced by perfectly competitive firms, which use capital and labor as inputs for production. Monopolistically competitive retailers buy a continuum of intermediate goods, and assemble them into a final good. Nominal frictions as in [Calvo \(1983\)](#) make the retailers optimization problem dynamic. Additionally, a capital producing sector buys up capital from the intermediate good producer, repairs it, and builds new capital, which it sells to the intermediate good sector again. Investment in new capital is subject to investment adjustment costs.

Intermediate Good Producers

In this setup dynamic pricing and investment decisions are carried out by retailers and capital good producers, respectively. Thus, the optimization of the intermediate good producers can be reduced to a sequence of static problems. Their production function takes a standard Cobb Douglas form, given by

$$Y_{mt} = A_t (\xi_t U_t K_{t-1})^\alpha L_t^{1-\alpha}, \quad (5)$$

where $0 < \alpha < 1$. A_t is an index for the level of technology, and U_t is the variable utilization rate of the installed capital. K_{t-1} is the capital purchased and installed in period $t-1$, which becomes productive in period t , and ξ_t is a shock to the quality of capital which can be interpreted as obsolescence of the employed capital. [Gertler and Kiyotaki \(2010\)](#) and [Gertler and Karadi \(2011\)](#) use this capital quality shock to simulate the banking crisis preceding the Great Recession in the US. In the context of this analysis it allows me to account for a source of risk for the price of capital assets.

At the end of each period the intermediate good producer sells the capital stock that it used for production to the capital producer which repairs the capital, and purchases the capital stock that it is going to use in the next period from the capital producer. To finance the purchase of the new capital at the price Q_t per unit, it issues a claim for each unit of capital it acquires to banks, which trade at the same price. The interest rate the firm has to pay on the loan from the bank is $R_{k,t}$. Under the assumption that the competitive firms make zero profits, the interest rate on their debt will just equal the realized ex-post return on capital. The resale value of the capital used in production depends on the realization of the capital quality shock, and the depreciation rate which in turn depends on the capital utilization rate in the following way

$$\delta(U_t) = \delta_c + \frac{b}{1+\zeta} * U_t^{1+\zeta}, \quad (6)$$

where δ_c and b are constants and ζ is the elasticity of the depreciation rate with respect to the utilization rate. Furthermore, capital evolves according to the following law of motion

$$K_t = (1 - \delta(U_t))\xi_t K_{t-1} + I_t. \quad (7)$$

Hence each period the firm in its investment decision maximizes

$$E_t[\beta\Lambda_{t,t+1}(-R_{k,t+1}Q_t K_t + P_{m,t+1}Y_{m,t+1} - W_{t+1}L_{t+1} + (1 - \delta(U_{t+1}))Q_{t+1}K_t\xi_{t+1})]$$

with respect to K_t . In optimum the ex-post return then is as follows

$$R_{k,t+1} = \frac{P_{m,t+1}\alpha\frac{Y_{m,t+1}}{K_t} + (1 - \delta(U_{t+1}))Q_{t+1}\xi_{t+1}}{Q_t}. \quad (8)$$

Additionally, the optimal choices of labor input and the capital utilization rate yield the first order conditions

$$W_t = P_{mt}(1 - \alpha)\frac{Y_{mt}}{L_t}, \quad (9)$$

$$\delta'(U_t)\xi_t Q_t K_{t-1} = P_{mt}\alpha\frac{Y_{mt}}{U_t}. \quad (10)$$

Capital Good Producers

The capital good producer's role in the model is to isolate the investment decision that becomes dynamic through the introduction of convex investment adjustment costs, which is a necessary feature to generate variation in the price of capital. Capital good producers buy the used capital, restore it and produce new capital goods. Since capital producers buy and sell at the same price, the profit they make is determined by the difference between the quantities sold and bought, i.e. investment. Thus they choose the optimal amount of investment to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_{0,t} \left\{ [(Q_t - 1)I_t - f\left(\frac{I_t}{I_{t-1}}\right)I_t] \right\}.$$

The first order condition of the capital producer reads

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}}f'\left(\frac{I_t}{I_{t-1}}\right) - E_t\beta\Lambda_{t,t+1}\left(\frac{I_t}{I_{t-1}}\right)^2 f'\left(\frac{I_t}{I_{t-1}}\right), \quad (11)$$

where the functional form of the investment adjustment costs is

$$f\left(\frac{I_t}{I_{t-1}}\right) = \frac{\eta_i}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2.$$

Retailers

Retailers produce differentiated goods by re-packaging the intermediate goods. They operate under monopolistic competition and face nominal rigidities à la [Calvo \(1983\)](#). As an additional element to smooth the equilibrium dynamics of inflation, it is assumed that in each period the fraction of firms that cannot choose its optimal price, γ , indexes its price to the inflation of the

foregoing period. The parameter of price indexation is γ_p .

Aggregate final output, Y_t , is described by a CES aggregator of the individual retailers' final goods, Y_{ft}

$$Y_t = \left(\int_0^1 Y_{ft}^{\frac{\epsilon-1}{\epsilon}} df \right)^{\frac{\epsilon}{\epsilon-1}}.$$

where $\epsilon > 1$ is the elasticity of substitution between different varieties of final goods. Thus the demand for its final goods that the retailer faces is

$$Y_{ft} = \left(\frac{P_{ft}}{P_t} \right)^{-\epsilon} Y_t,$$

where P_{ft} is the price chosen by retailer f . The aggregate price index is

$$P_t = \left(\int_0^1 P_{ft}^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}},$$

which due to the specific assumptions on the nominal rigidity can be written as

$$\Pi_t^{1-\epsilon} = (1-\gamma)(\Pi_t^*)^{1-\epsilon} + \gamma \Pi_{t-1}^{\gamma_p(1-\epsilon)}, \quad (12)$$

where $\Pi_t := \frac{P_t}{P_{t-1}}$, and $\Pi_t^* := \frac{P_t^*}{P_{t-1}}$. As the retailers' only input is the intermediate good which is sold by competitive producers, the marginal cost of the retailers equals the price of the intermediate good. Hence, each retailer chooses its optimal price to maximize the sum of its expected discounted profits

$$E_t \sum_{i=0}^{\infty} (\gamma\beta)^i \Lambda_{t,t+i} \left\{ \frac{P_t^*}{P_{t+i}} \prod_{k=1}^i (\Pi_{t+k-1})^{\gamma_p} - P_{m,t+i} \right\} Y_{f,t+i},$$

subject to the demand constraint. The first order condition for optimal price setting reads

$$E_t \sum_{i=0}^{\infty} (\gamma\beta)^i \Lambda_{t,t+i} \left\{ \frac{P_t^*}{P_{t+i}} \prod_{k=1}^i (\Pi_{t+k-1})^{\gamma_p} - \frac{\epsilon-1}{\epsilon} P_{m,t+i} \right\} \left(\frac{P_t^*}{P_{t+i}} \right)^{-\epsilon} Y_{t+i} = 0.$$

Accordingly, the optimal choice of the price implies:

$$\Pi_t^* = \frac{\epsilon}{\epsilon-1} \frac{F_t}{Z_t} \Pi_t, \quad (13)$$

where F_t and Z_t are defined recursively as

$$F_t = Y_t P_{mt} + \beta\gamma \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon} \Pi_t^{-\gamma_p \epsilon} F_{t+1}, \quad (14)$$

$$Z_t = Y_t + \beta\gamma \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon-1} \Pi_t^{-\gamma_p(\epsilon-1)} Z_{t+1}. \quad (15)$$

Equations (13)-(16) constitute the equilibrium conditions which in a linearized form are equivalent to a New Keynesian Phillips Curve with price indexation. Aggregate output of final goods, Y_t , is related to the aggregate intermediate output, Y_{mt} , in the following way

$$Y_{mt} = \Delta_{p,t} Y_t, \quad (16)$$

where Δ_t is the dispersion of individual prices, which evolves according to the law of motion

$$\Delta_{p,t} = \gamma \Delta_{p,t-1} \Pi_t^{\epsilon} \Pi_{t-1}^{-\gamma_p \epsilon} + (1-\gamma) \left(\frac{1 - \gamma \Pi_t^{\epsilon-1} \Pi_{t-1}^{-\gamma_p(\epsilon-1)}}{1-\gamma} \right)^{\frac{\epsilon}{\epsilon-1}}. \quad (17)$$

The markup X_t of the monopolistic retailers is the inverse of their marginal costs, which is equivalent to the price of the intermediate good

$$X_t = \frac{1}{P_{mt}}. \quad (18)$$

Banks

Banks finance their operations by creating deposits, D_t , which are held by households, and by their net worth, N_t . They use their funds to extend loans to intermediate good producers for acquiring capital, K_t , and for the purchases of government bonds, B_t at their market price Q_t^b . The balance sheet of bank j is given by

$$Q_t K_{jt} + Q_t^b B_{jt} = N_{jt} + D_{jt}. \quad (19)$$

The banks retain the earnings, generated by the return on their assets purchased in the previous period, and add it to their current net worth. Thus, the law of motion for the net worth of a bank is given by

$$N_{jt} = R_{kt} Q_{t-1} K_{j,t-1} + R_{bt} Q_{t-1}^b B_{j,t-1} - R_{t-1} D_{j,t-1}. \quad (20)$$

Note that while the interest rate on deposits raised in period $t - 1$, is determined in the same period, the return of the risky capital assets and risky government bonds purchased in period $t - 1$ is determined only after the realization of shocks at the beginning of period t . Substituting the balance sheet into the law of motion for net worth yields

$$N_{jt} = (R_{kt} - R_{t-1}) Q_{t-1} K_{j,t-1} + (R_{bt} - R_{t-1}) Q_{t-1}^b B_{j,t-1} + R_{t-1} N_{j,t-1}. \quad (21)$$

Bankers continue accumulating their net worth, until they exit the business. Each period, each banker faces a lottery, which determines, regardless of the history of the banker, whether he exits his business or stays in the sector. Bankers exit the business with an exogenous probability $1 - \theta$, or continue their operations with probability θ . The draws of this lottery are iid. When a banker leaves the sector, it adds his terminal wealth V_t to the wealth of its household. Therefore, bankers seek to maximize the expected discounted terminal value of their wealth

$$\begin{aligned} V_{jt} &= \max E_t \sum_{i=0}^{\infty} (1 - \theta) \theta^i \beta^{i+1} \Lambda_{t,t+1+i} N_{j,t+1+i} \\ &= \max E_t [\beta \Lambda_{t,t+1} (1 - \theta) N_{j,t+1} + \theta V_{j,t+1}]. \end{aligned}$$

As banks operate under perfect competition, with perfect capital markets the risk adjusted return on loans and government bonds would equal the return on deposits. However, bankers face an endogenous limit on the amount of funds that households are willing to supply as deposits. Following [Gertler and Karadi \(2011\)](#), I assume that bankers can divert a fraction of their assets and transfer it to their respective households. However, if they do so, their depositors will choose to withdraw their remaining funds and force the bank into bankruptcy. To avoid this scenario, households will keep their deposits at a bank only as long as the bank's continuation value is higher or equal to the amount that the bank can divert. Formally, the incentive constraint of the bank reads

$$V_{jt} \geq \lambda Q_t K_{jt} + \lambda_b Q_t^b B_{jt}, \quad (22)$$

where λ , is the fraction of loans that the bank can divert, and λ_b is the fraction of government bonds it can divert. I calibrate λ_b to be smaller than λ . This is motivated by the fact, that, in general, the collateral value of government bonds is higher than that of loans.⁶ The reason is that loans to private firms are less standardized than government bonds contracts. Additionally, information on the credit-worthiness of the government is publicly available, while the credit-worthiness of private firms is often only known to the bank and the firm, and not easy to assess for depositors, making it easier for banks to divert a fraction of their value.

⁶This is in the vein of Meeks et al. (2014), who use the same approach to distinguish between the collateral values of loans and asset backed securities.

The initial guess for the form of the value function is

$$V_{jt} = v_{kjt}Q_tK_{jt} + v_{bjt}Q_t^bB_{jt} + v_{njt}N_{jt}, \quad (23)$$

where v_{kjt} , v_{bjt} and v_{njt} are time varying coefficients. Maximizing (23) with respect to loans and bonds, subject to (22) yields the following first order conditions for loans, bonds, and μ_t , the Lagrangian multiplier on the incentive constraint

$$v_{kjt} = \lambda \frac{\mu_{jt}}{1 + \mu_{jt}}, \quad (24)$$

$$v_{bjt} = \lambda_b \frac{\mu_{jt}}{1 + \mu_{jt}}, \quad (25)$$

$$v_{kjt}Q_tK_{jt} + v_{bjt}Q_t^bB_{jt} + v_{njt}N_{jt} = \lambda Q_tK_{jt} + \lambda_b Q_t^bB_{jt}. \quad (26)$$

Given that the incentive constraint binds⁷, a bank's supply of loans can be written as

$$Q_tK_{jt} = \frac{v_{bjt} - \lambda_b}{\lambda - v_{kjt}} Q_t^bB_{jt} + \frac{v_{njt}}{\lambda - v_{kjt}} N_{jt}. \quad (27)$$

As (27) shows, the supply of loans decreases with an increase in λ , which regulates the tightness of the incentive constraint with respect to capital, and increases with an increase in λ_b , which makes the holding of bonds more costly in terms of a tighter constraint. Plugging the demand for loans into (23), and combining the result with (24) and (25) one can write the terminal value of the banker as a function of its net worth⁸

$$V_{jt} = (1 + \mu_{jt})v_{njt}N_{jt}. \quad (28)$$

A higher continuation value, V_{jt} is associated with a higher shadow value of holding an additional marginal unit of assets, or put differently, with a higher shadow value of marginally relaxing the incentive constraint. Defining the stochastic discount factor of the bank to be

$$\Omega_{j,t} \equiv \Lambda_{t-1,t}((1 - \theta) + \theta(1 + \mu_{jt})v_{njt}), \quad (29)$$

plugging (28) into the Bellman equation, and using the law of motion for net worth, one can then write the value function as

$$\begin{aligned} V_{jt} &= E_t[\beta\Lambda_{t,t+1}(1 - \theta)N_{j,t+1} + \theta V_{j,t+1}] \\ &= E_t[\beta\Omega_{j,t+1}((R_{k,t+1} - R_t)Q_tK_{j,t} + (R_{b,t+1} - R_t)Q_t^bB_{j,t} + R_tN_{j,t-1})], \end{aligned}$$

and verify the initial guess for the value function as

$$v_{kjt} = \beta E_t \Omega_{j,t+1} (R_{k,t+1} - R_t), \quad (30)$$

$$v_{bjt} = \beta E_t \Omega_{j,t+1} (R_{b,t+1} - R_t), \quad (31)$$

$$v_{njt} = \beta E_t \Omega_{j,t+1} R_t. \quad (32)$$

⁷The constraint binds in the neighborhood of the steady state. For convenience, I make the assumption that it is binding throughout all experiments.

⁸Detailed derivations are delegated to the appendix.

Aggregation of financial variables

To facilitate aggregation of financial variables, I assume that banks share the same structure to the extent that they derive the same respective values from holding loans and bonds, and from raising deposits (i.e., $\forall j : v_{kjt} = v_{kt}, v_{bjt} = v_{bt}, v_{njt} = v_{nt}$). Furthermore, I assume that all banks have the same ration of capital assets to government bonds, $\zeta_t \equiv \frac{Q_t K_t}{Q_t^b B_t}$, in their portfolio. As an implication, the leverage ratio of banks does not depend on the conditions that are specific to individual institutes, and all banks share the same weighted leverage ratio⁹

$$\phi_t \equiv \frac{v_{nt}(1 + \zeta_t)}{(\lambda - v_{kt})(1 + \frac{\lambda_b}{\lambda} \zeta_t)} = \frac{Q_t K_t + Q_t^b B_t}{N_t}.^{10} \quad (33)$$

Note that the lower divertability of government bonds relative to capital assets, allows the bank to increase its leverage ratio, compared to a scenario in which banks only hold capital assets. The aggregate balance sheet constraint reads

$$Q_t K_t + Q_t^b B_t = D_t + N_t. \quad (34)$$

The net worth of the fraction of bankers that survive period $t - 1$ and continue operating in the banking sector, θ , can be written as

$$N_{ot} = \theta \left[R_{kt} Q_{t-1} K_{t-1} + R_{bt} Q_{t-1}^b B_{t-1} - R_{t-1} D_{t-1} \right]. \quad (35)$$

A fraction $(1 - \theta)$ of bankers leaves the business. There is a continuum of bankers, and the draws out of the lottery, which determines whether a banker stays in business or exits the sector, are iid. Hence, by the law of large numbers, it follows that the share of assets that leaves the sector is a fraction $(1 - \theta)$ of the total assets. At the same time, new bankers enter the sector. New bankers are endowed with "start-up funding" by their households. The initial endowment of the new bankers is proportionate to the assets that leave the sector. The net worth of the new bankers, N_{nt} , can be written as

$$N_{nt} = \omega \left[Q_{t-1} K_{t-1} + Q_{t-1}^b B_{t-1} \right], \quad (36)$$

where ω is calibrated to ensure that the size of the banking sector is independent of the turnover of bankers. Aggregate net worth, N_t , is then the sum of the net worth of old and new bankers

$$N_t = N_{ot} + N_{nt}. \quad (37)$$

Fiscal Policy

The fiscal sector follows closely the structure in [van der Kwaak and van Wijnbergen \(2013\)](#) and [van der Kwaak and van Wijnbergen \(2015\)](#). The government finances its expenditures, G_t , by issuing government bonds, which are bought by banks, and by raising lump sum taxes, T_t . Government spending is exogenous and follows an AR(1) process

$$G_t = G e^{g_t}, \quad (38)$$

$$\text{and } g_t = \rho_g g_{t-1} + \epsilon_t^g, \quad (39)$$

⁹Details are delegated to the appendix.

¹⁰Note that if the collateral values of capital assets and lambda were the same ($\lambda = \lambda_b$), the leverage ratio would take the same form as in [Gertler and Karadi \(2011\)](#), or in [Kirchner and van Wijnbergen \(2016\)](#)

where G is the steady state government consumption, ρ_g is the autocorrelation of government consumption, and ϵ_t^g is a shock to government spending. Taxes follow a simple feedback rule, such that they are sensitive to the level of debt and to changes in government expenditures

$$T_t = T + \kappa_b(B_{t-1} - B) + \kappa_g(G_t - G), \quad (40)$$

where T and B are the steady state levels of tax revenue and government debt, respectively. κ_b is set to ensure that the real value of debt grows a rate smaller than the gross real rate on government debt. As shown by [Bohn \(1998\)](#), this rule is a sufficient condition to guarantee the solvency of the government. If κ_g is set to zero, increases in government expenditures are entirely debt-financed. In turn, when $\kappa_g = 1$, changes in government spending are tracked one-to-one by changes in taxes.

To allow for the calibration of a realistic average maturity of government debt, bonds are modeled as consols with geometrically decaying coupon payments, as in [Woodford \(1998\)](#) and [Woodford \(2001\)](#). A bond issued in period t at the price of Q_t^b , pays out a coupon of r_c in period $t + 1$, a coupon of $\rho_c r_c$ in period $t + 2$, a coupon of $\rho_c^2 r_c$ in $t + 3$, and so on. Setting the decay factor ρ_c equal to zero captures the case of a one-period bond, in which the entire payoff of the bond is due in period $t + 1$. Setting $\rho_c = 1$ delivers the case of a perpetual bond. The average maturity of a bond of this type is $1/(1 - \beta\rho_c)$. For investors, this payoff structure is equivalent to receiving the coupon r_c and a fraction, ρ_c , of a similarly structured bond in period $t + 1$. The beginning-of-period debt of the government can thus be summarized as $(r_c + \rho_c Q_t^b)B_{t-1}$.

At the beginning of each period, the government has the option to default and write off a fraction of its debt, $D \in (0, 1)$. Investors take this into account, and demand a higher return on government bonds, when the expected probability of a sovereign default, Δ_{t+1}^d , increases. The return to government bonds, adjusted for default risk, can thus be written as

$$R_{b,t} = (1 - \Delta_t^d * D) \left[\frac{r_c + \rho_c Q_t^b}{Q_{t-1}^b} \right]. \quad (41)$$

The flow budget constraint of the government reads

$$\begin{aligned} G_t + Q_t^b B_t &= R_{bt} Q_{t-1}^b B_{t-1} + T_t, \\ \text{or: } G_t + Q_t^b B_t &= (1 - \Delta_t^d * D) \left[\frac{r_c + \rho_c Q_t^b}{Q_{t-1}^b} \right] Q_{t-1}^b B_{t-1} + T_t. \end{aligned} \quad (42)$$

Linking the probability of a sovereign default to the level of public debt or the debt-to-GDP ratio is common in the literature (see, e.g., [Eaton and Gersovitz \(1981\)](#), [Arellano \(2008\)](#) or [Leeper and Walker \(2011\)](#)). A higher level of public debt implies a higher debt service, and, in turn, requires higher tax revenues to service the interest rate payments. As tax increases are not popular and only up to a maximum level politically feasible, it is plausible to posit a maximum capacity of levying taxes, or fiscal limit. With an increasing public debt, the economy moves closer to the fiscal limit.

The probability of a sovereign default is described by the logistical distribution function

$$\Delta_t^d = \frac{\exp\left(\eta_1 + \eta_2 \frac{B_t}{4Y_t}\right)}{1 + \exp\left(\eta_1 + \eta_2 \frac{B_t}{4Y_t}\right)}, \quad (43)$$

which depends on the debt-to-GDP ratio, and is depicted in figure (2). The fiscal limit function is pinned down by the parameters η_1 and η_2 . I use the results of the structural estimation of an RBC model on Italian data by [Bi and Traum \(2012a\)](#) to calibrate these parameters.

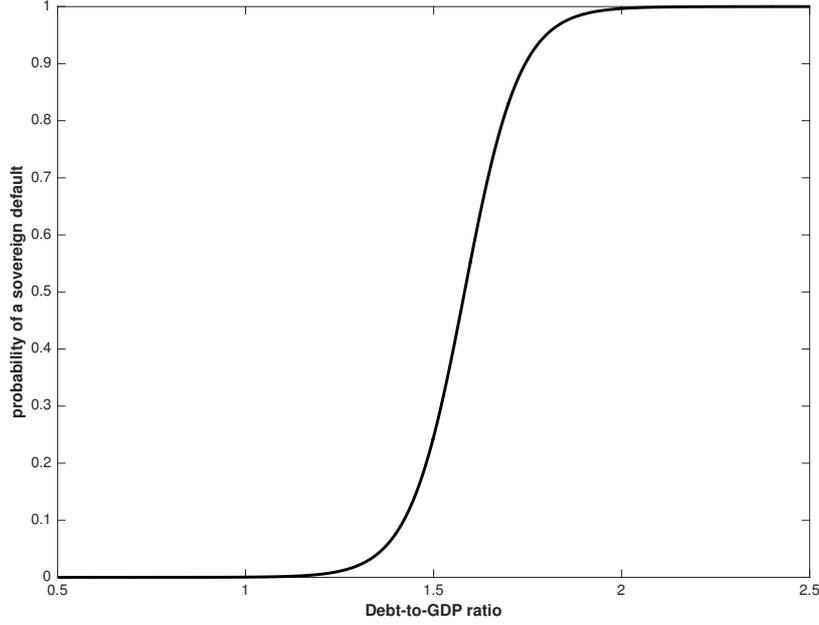


Figure 2: Default indicator, Δ_t^d

Monetary Policy and Good Market Clearing

The policy tool of the central bank in this economy is the nominal interest rate, i_t , which is set to the Taylor-type rule

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)(i + \kappa_\pi \pi_t + \kappa_y \hat{m}c_t) + \epsilon_t^i, \quad (44)$$

where the smoothing parameter ρ_i lies between zero and one, ϵ_t^i is a monetary policy shock, and $\kappa_\pi > 1$ to satisfy the Taylor principle and guarantee the determinacy of the rational expectation equilibrium. The real interest rate on deposits and the nominal policy rate of the central bank are linked via the Fisher equation

$$1 + i_t = R_t \frac{E_t P_{t+1}}{P_t}. \quad (45)$$

Finally, the good market clears

$$Y_t = C_t + I_t + f\left(\frac{I_t}{I_{t-1}}\right) I_t + G_t. \quad (46)$$

The equilibrium conditions of the full model are collected in the appendix.

3. Calibration and Solution Method

Calibration

The calibration of the model is motivated by the case of the Italian economy, which represents a case of a large, relatively closed economy with high public debt, and recurring periods of high interest rate spreads in the last decades. The model is calibrated to quarterly frequency. Table 1 lists the parameter values used in the model. A major source for the parameter values is [Bi and Traum \(2012a\)](#). [Bi and Traum \(2012a\)](#) estimate an RBC model with a sovereign default indicator

on Italian data from 1999.Q1 and 2010.Q3. As values for the discount factor, β , and the coefficient of relative risk aversion they choose 0.99 and 1.00, respectively. In addition, they choose for the deterministic steady state a debt-to-GDP ratio of 1.19, and an output share of government spending of 0.1966 to match the respective means in their sample. In accordance with their estimation results, I set the parameter for consumption habit, h , to 0.14, the persistence and standard deviation of the technology shock, ρ_a and σ_a to 0.96 and 0.01, and the persistence and standard deviation of the government spending shock, ρ_g and σ_g to 0.84 and 0.01, respectively. Furthermore they obtain the values 0.3 for κ_b and 0.53 for κ_g . Under the assumption of a haircut of 37.8 percent on the outstanding debt in case of a sovereign default, their estimation results imply a default function with the parameter values $\eta_1 = -21.5285$ and $\eta_2 = 3.4015$.¹¹

The values of the parameters of the default function reflect that the estimation by [Bi and Traum \(2012a\)](#) is based on a sample that largely contains years, in which the market for Italian government bond was calm. They imply a quarterly default rate of 0.0048 on sovereign bonds, and a low sensitivity of the default rate to movements in the debt-to-GDP ratio at the deterministic steady state. Similarly, the calibration of the parameters associated with the government spending shock and the technology shock reflects a mixture of calm years and crisis years. I choose these parameter values by [Bi and Traum \(2012a\)](#) as my benchmark calibration, since an empirically plausible calibration of the default function is crucial for the assessment of the role of fiscal stress for the multiplier. Also, an empirically plausible calibration of the shock processes is important for the assessment of the role of aggregate risk for the dynamics of the model at a third-order approximation. In section 4.3., however, I deviate from the benchmark calibration, and analyze, among others, how the results are affected, when the calibration is adjusted such as to match features of the recent debt crisis in the Italy, in which the debt-to-GDP ratio rose to levels of 1.3 and arguably the sensitivity of fiscal stress to changes in the debt-to-GDP ratio, and the degree of aggregate risk in the economy may have been higher.¹²

In the calibration of the parameters associated with the different firm sectors, I borrow from the model by [Gertler and Karadi \(2011\)](#), which is the fundament of the framework at hand.¹⁴ The effective capital share, α is 0.33. In the deterministic steady state, the utilization rate of capital is 1, and the elasticity of marginal depreciation with respect to capital utilization, ζ , is 7.2. The parameters b and δ_c are chosen such as to yield a depreciation rate of 0.025 in the deterministic steady state. Parameter η_i , which governs the investment adjustment costs is set to 1.72 and the elasticity of intra-temporal substitution, η , is set to 4.167. The Calvo parameter, $\gamma = 0.779$ implies an average price duration of roughly four and a half quarters. The degree of price indexation, γ_p is rather low at 0.241.

Further parameter values that I use from [Gertler and Karadi \(2011\)](#), are the inverse of the Frisch elasticity, φ , the feedback parameters in the Taylor rule, κ_π and κ_y , the persistence of the capital quality shock ρ_ξ , and some parameters of the banking sector. In the calibration of the banking sector, I follow a similar strategy as [Gertler and Karadi \(2011\)](#), and choose the fractions of the assets that the banks can diverge, λ and λ_b , the survival probability of the bankers, θ , and the transfer to new bankers, ω , in order to target a steady state leverage ratio of 4, an average time horizon of the bankers of a decade, and the steady state spreads of the returns on the banks assets over the deposit rate. For the steady state spread of the return on capital over deposits I use the same target as in [Gertler and Karadi \(2011\)](#). For the steady state spread of government bonds over deposits I take the estimate by [Bocola \(2015\)](#) as a guideline. The difference of the steady state spreads of the two assets is reflected by the values of the respective divertability parameters, λ

¹¹For more details on the estimation procedure, see [Bi and Traum \(2012a\)](#)

¹²The literature on stochastic volatility¹³ argues that the volatility of real and financial variables increases during crisis times.

¹⁴In turn, [Gertler and Karadi \(2011\)](#) borrow most of their parameter values from [Primiceri, Schaumburg, and Tambalotti \(2006\)](#). who estimate a medium scale model on US data.

and λ_b . In the deterministic steady state these two parameters are linked by the relation:

$$\frac{\lambda}{\lambda_b} = \frac{R_k - R}{R_b - R}.$$

The coupon rate on the long-term government bond, r_c , is set to 0.04, and the rate of decay of the bonds, ρ_c , is set to 0.96 as in van der Kwaak and van Wijnbergen (2015). The standard deviations of the monetary policy shock and the capital quality shock are set to 0.01. The parameter χ , which weighs the disutility of labor is chosen such as to balance the labor supply equation in the deterministic steady state.

β	discount factor	0.99	Bi and Traum (2012)
h	habit formation	0.14	Bi and Traum (2012)
χ	weight for disutility of labor	4.7125	
φ	inverse of Frisch elasticity	0.276	Gertler and Karadi (2011)
α	effective capital share	0.33	Gertler and Karadi (2011)
ζ	cap. util. param.	7.2	Gertler and Karadi (2011)
b	cap. util. param.	0.03760101	Gertler and Karadi (2011)
δ_c	depreciation param.	0.020414511	Gertler and Karadi (2011)
η_i	invest. adjust. param.	1.728	Gertler and Karadi (2011)
ϵ	elasticity of substitution	4,167	Gertler and Karadi (2011)
γ	Calvo param.	0.779	Gertler and Karadi (2011)
γ_p	price indexation param.	0.241	Gertler and Karadi (2011)
$Rk - R$	steady state spread	0.01	Gertler and Karadi (2011)
$Rb - R$	steady state spread	0.005	Bocola (2015)
λ	divertibility of capital assets	0.4479	
λ_b	divertibility of bonds	0.2239	
θ	survival probability of banker	0.975	Gertler and Karadi (2011)
ω	transfer to new bankers	0.0018	
κ_π	interest rate rule	1.5	Gertler and Karadi (2011)
κ_y	interest rate rule	-0.125	Gertler and Karadi (2011)
G/Y	share of gov. spending	0.1966	Bi and Traum (2012)
$B/4Y$	debt-to-GDP ratio.	1.19	Bi and Traum (2012)
κ_b	tax rule param.	0.3	Bi and Traum (2012)
κ_g	tax rule param.	0.53	Bi and Traum (2012)
η_1	fiscal limit parameter	-21.5285	Bi and Traum (2012)
η_2	fiscal limit parameter	3.4015	Bi and Traum (2012)
D	haircut	0.378	Bi and Traum (2012)
r_c	coupon rate	0.04	v.d.Kwaak and v. Wijnbergen (2014)
ρ_c	rate of decay of consol	0.96	v.d.Kwaak and v. Wijnbergen (2014)
ρ_i	intrest rate smoothing	0.0	v.d.Kwaak and v. Wijnbergen (2014)
ρ_ξ	persistence of ξ -shock	0.66	Gertler and Karadi (2011)
ρ_a	persistence of a-shock	0.96	Bi and Traum (2012)
ρ_g	persistence of g-shock	0.84	Bi and Traum (2012)
σ_i	std. of i-shock	0.01	
σ_ξ	std. of ξ -shock	0.01	
σ_g	std. of g-shock	0.01	Bi and Traum (2012)
σ_a	std. of a-shock	0.01	Bi and Traum (2012)

Table 1: Calibration of Parameters

Solution method

The model is solved using a third-order approximation to equilibrium dynamics. I employ the algorithm developed by [Lan and Meyer-Gohde \(2013\)](#).¹⁵ Their solution method solves for the policy functions in the form of a nonlinear moving average. Let the nonlinear DSGE model be

$$E_t f(y_{t+1}, y_t, y_{t-1}, \epsilon_t) = 0, \quad (47)$$

where y_t and ϵ_t represent the vectors of endogenous variables and the vector of exogenous shocks, respectively. Then the solution to this model can be written as a system of policy functions of the form:

$$y_t = y(\sigma, \epsilon_t, \epsilon_{t-1}, \epsilon_{t-2}, \dots), \quad (48)$$

where σ scales the degree of aggregate risk of the model.¹⁶ Under the assumption of normally distributed shocks (i.e., with zero skewness), the third-order Taylor approximation of the policy function takes the form:

$$y_t = \bar{y} + \frac{1}{2} y_{\sigma^2} \sigma^2 + \sum_{i=0}^{\infty} \left(y_i + \frac{1}{2} y_{\sigma_i^2} \sigma^2 \right) \epsilon_{t-i} + \frac{1}{2} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} y_{i,j} (\epsilon_{t-i} \otimes \epsilon_{t-j}) + \frac{1}{6} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} y_{i,j,k} (\epsilon_{t-i} \otimes \epsilon_{t-j} \otimes \epsilon_{t-k}), \quad (49)$$

where \bar{y} denotes the vector of deterministic steady state values of the respective endogenous variables, and the partial derivatives $y_i, y_{i,j}, y_{i,j,k}, y_{\sigma^2}$ and $y_{\sigma_i^2}$ are evaluated at the deterministic steady state. Up to the first order approximation, the policy function is independent of the degree of risk in the model. However, when the model is approximated with a third-order approximation, two terms enter the policy function that adjust it for the risk of future shocks. While y_{σ^2} is constant, $y_{\sigma_i^2}$ varies over time and interacts the linear impulse responses to realized shocks with the risk of future shocks.

In the case at hand, this becomes particularly relevant as leverage constrained banks hold risky government bonds on their balance sheets. To the extent that government spending shocks affect prices and returns of the banks' assets, they imply a time-varying adjustment of the banks' asset holdings to aggregate risk, which alters the reaction of output to the shock. As the economy moves closer to the fiscal limit the default probability becomes more sensitive to the fluctuations in the debt-to-GDP ratio,¹⁷ Therefore, the fiscal stress channel amplifies this time-varying adjustment to risk which, as I will show, affects the dynamic consequences of government spending shocks to output in a quantitatively relevant way, and thus requires a third-order approximation of equilibrium dynamics.¹⁸

¹⁵[Lan and Meyer-Gohde \(2013\)](#) show that for the nonlinear moving average method it holds that, if the first-order solution is stationary and saddle-stable, these properties carry over to higher-order approximation.

¹⁶ $\sigma = 0$ corresponds to the non-stochastic model, whereas $\sigma = 1$ corresponds to the model with the originally assigned distribution of shocks.

¹⁷This corresponds to the empirical finding of a higher sensitivity of default risk premia to fundamentals in fiscal crises as found by [Beirne and Fratzscher \(2013\)](#). [Beirne and Fratzscher \(2013\)](#) label this as an increase in "fundamental awareness".

¹⁸Note, that while in principle, time varying risk-adjustment affects the dynamic consequences of level shocks in any DSGE model, the risk-adjustment components are in general of negligible size. For the neoclassical growth model, this is illustrated by [Lan and Meyer-Gohde \(2013\)](#)

4. Dynamic Analysis of government spending shocks

This section analyses the effects of a shock to government spending. In order to illustrate the effects of the fiscal stress channel for the output effects of the shock, I compare the full two model described in section 2. with a model, in which the fiscal stress channel is shut of. Here, I eliminate equation (43) from the model, and set variable Φ_t is to a constant (i.e., $\Phi_t = \Phi \ \forall t$).¹⁹ In all other aspects the two models are identical. I label the latter model: model without fiscal stress .

4.1. The linearized model

Let me first focus on the analysis of the impulse responses to a government spending shock in the linearized model. The shock is of the size of 1 percent of steady state GDP. Figure (3) depicts the impulse responses. The blue lines show the effect of the shock in the benchmark model with fiscal stress , whereas the black lines show the effect of the shock in a model, which abstracts from the role of fiscal stress for the transmission of the shock. As figure (3) illustrates the contribution of fiscal stress is negligible in the linearized case. However, before I turn to the non-linear case, I briefly discuss in this section how the features embedded in the model affect the dynamic consequences of the government spending shock.

As the model contains several features, which affect the transmission of the shock, I start with a brief description of those effects that are independent of financial frictions or fiscal stress. The government spending shock increases government debt and stimulates output. Private activity is crowded out, however. Since households are Ricardian, and preferences are additive separable in consumption and leisure, the increase in government spending induces a wealth effect on the labor supply.²⁰ As in many models, consumption falls and the labor supply increases after the shock, leading to increasing equilibrium labor hours and a decline in the real wage. With the higher labor input the marginal product of labor decreases. In the presence of sticky prices, the average real marginal cost increase and the average markup declines with the increase in production. As a result of the falling markup, the demand for labor increases for any given real wage, contributing to the increase in equilibrium labor hours and output.²¹ As highlighted by [Basu and Kimball \(2003\)](#), the increase in labor leads to an increase in the marginal product of capital and thus to a higher real return on capital. This has the effect that investment becomes less attractive, leading to a decline in the capital stock as well as in the price of capital. The firms' demand for loans from banks contracts.

The resulting decline of investment in response to the increase in government spending, that is essential for the link between fiscal stress and the multiplier in my model, is consistent with empirical evidence. For the US, a decline in investment in response to the shock can be found across different approaches to the identification of government spending shocks (see, e.g., [Blanchard and Perotti \(2002\)](#), [Mountford and Uhlig \(2009\)](#), [Ramey \(2011\)](#)). Evidence from cross-country panels provided by [Corsetti et al. \(2012\)](#) and by [Ilzetzki et al. \(2013\)](#) similarly points to a decline in investment after an increase in government spending.

¹⁹The value of Φ_t in the deterministic steady state, is the same in both models.

²⁰For a discussion of the wealth effect on the labor supply, see: e.g., [Christiano and Eichenbaum \(1992\)](#) or [Baxter and King \(1993\)](#).

²¹For an analysis of the role of markup shifts for the transmission of government spending shocks in the New Keynesian model, see: [Monacelli and Perotti \(2008\)](#).

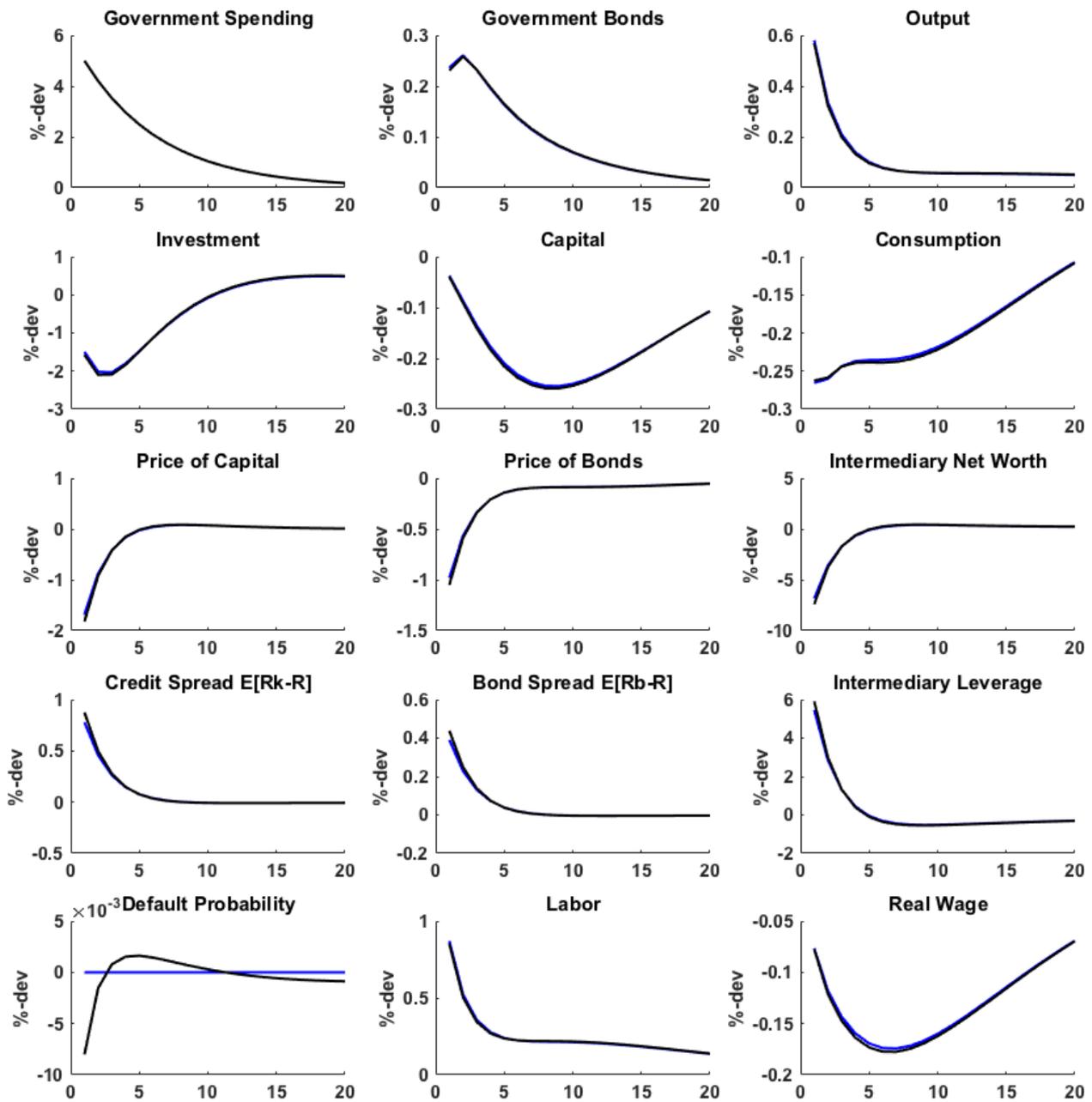


Figure 3: Dynamic consequences of a shock in government spending of 1 percent obtained from a first-order approximation to equilibrium dynamics. The blue lines depict the impulse responses of a model without fiscal stress. The black line depicts the impulse responses with fiscal stress.

As [Kirchner and van Wijnbergen \(2016\)](#) show, the crowding-out of investment is amplified through a contraction in the supply of loans, if government bonds are held by leverage constrained as it is the case in this model. Banks reduce the supply of loans for two reasons: First, as they incur capital losses due to the decline in capital prices, their net worth shrinks, tightening their leverage constraints. Additionally, the price of government bonds falls together with the price of capital, due to arbitrage on financial markets, further tightening the leverage constraint. The fall in the prices of capital assets and bonds drive up the spread of the return on bonds over the return on deposits. Lastly, the reduction of the supply of loans, increases expected future returns on loans. Hence, the spread of the return on capital assets (loans from the banks to the firms), over the return of deposits increases. As a consequence of the falling asset prices, banks are forced to reduce the size of their balance sheets and sell capital assets, amplifying the fall in the price of capital, and to reduce the supply of loans. A second reason reason for the contraction of the loan supply is that banks change the composition of their asset portfolio. The increase in public debt increases their holdings of government bonds. Given the constraint on the size of banks' balance sheets, the portfolio shift towards government bonds exerts further pressure on the supply of loans to firms. Overall, the financial accelerator mechanism that is implied by constrained banks amplifies the crowding out of investment compared to a situation with unconstrained investors in capital assets, and lowers the government spending multiplier.

Now let me turn to the effect of fiscal stress on the government spending multiplier. Most of the literature on the interaction of fiscal multipliers and fiscal stress, theoretical as well as empirical, focusses on the possibility that higher fiscal stress decreases the impact of government spending shocks. Theoretical studies that focus on the effect of fiscal retrenchment suggest that, for instance, an improvement of the expectations of future growth ([Bertola and Drazen \(1993\)](#)), or a decrease in risk premia on government bonds, enhanced financial intermediation and increasing investment (e.g. [Alesina and Perotti \(1997\)](#), [Corsetti et al. \(2013\)](#) or [van der Kwaak and van Wijnbergen \(2013\)](#)) counteract the otherwise detrimental effects of fiscal retrenchment on output growth. I follow the authors who focus in their argument on the channel of financial intermediation and changes in risk premia.

The response to the government spending shock in the linearized model with fiscal stress depicted as black curves in figure (3). As one can see, the contribution of the fiscal stress channel to the response of the financial and real variables to the government spending shock is very small. The main difference to the scenario without fiscal stress is that now the default probability responds to the movements in the debt-to-GDP ratio. As the initial increase in GDP is larger in percentage terms than the initial increase in public debt, the debt-to-GDP ratio decreases on impact. This initial fall in the default probability after a fiscal expansion is in line with empirical findings by [Born et al. \(2015\)](#) and [Strobel \(2016\)](#) who find that in periods of a higher of fiscal stress, bond yield spreads increase on impact after contractionary fiscal shocks. Already after the second quarter, the default probability in the model rises above the steady state, since the increase in debt is more persistent than the output response. Through equation (43) the variation in the default probability contributes to the fall in price of bonds and the increase in the return on bonds, and as prices and returns of assets move together, it contributes to the fall in the price of capital and the increase in the return on capital as well. The contraction of investment and capital are slightly stronger, and the response of output slightly weaker than in the model without fiscal stress.

The size of this effect is, however, very small. Table 1 shows the difference in the government spending multipliers for the two models. Adding the feature of fiscal stress decreases the impact multiplier from 0.58 to 0.57, and the cumulative multiplier over a time horizon of 20 quarters

from 0.37 to 0.36. Thus, the linearized model cannot generate the empirical finding that the presence of fiscal stress lowers the government spending multiplier.

Table 1: Government Spending Multipliers

	Impact Multiplier	Cumulative Multiplier*
linear , no fiscal stress	0.581	0.368
linear , with fiscal stress	0.569	0.362

*time horizon of 20 quarters

4.2. The role of risk for the fiscal stress channel

This section discusses the role of risk for the transmission of government spending shocks. Figure (4) compares the linear impulse responses to the government spending shock in the full model with fiscal stress (dashed line) with the impulse responses to the same shock, obtained by a third-order approximation to equilibrium dynamics (solid line).

The main difference in the two sets of impulse responses lies with the time-varying adjustment for the risk of future shocks.²² The risk associated with the realization of future shocks matters because agents are risk-averse and their behavior now reflects a precautionary motive. In particular, the behavior of banks matters for the transmission of the shock. As in the linearized model, the increase in government spending crowds out investment and results in a decrease in asset prices and fire sales of assets by banks, who incur losses, and face a shrinking net worth and an increasing leverage ratio. Now, that banks take into account the risk of further, potentially detrimental, shocks in the future, they react more sensitive to changes in asset prices and interest rates, and they reduce their exposure to risky assets to a larger extent in response to the shock. This amplifies the fall in the prices of bonds and capital assets, and leads to a stronger contraction of the net worth of bankers. As the implied increase in banks' leverage is stronger, they further have to increase their fire sales and the contraction in the supply of loans is more pronounced. Hence, the fall in investment is stronger than in the linearized model, up to three percent, compared to a decline of roughly two percent in the linear case. Both values are well within range of the empirically observed decline after an identified government spending shock.²³

Importantly, risk also increases the role of fiscal stress for the multiplier. The steeper fall in investment leads to a smaller and more short-lived output stimulus in response to the increase in government spending. Four quarters after the shock, output even declines below the initial state, and the economy enters a mild recession. As the response of public debt is hardly altered, the initial fall in the debt-to-GDP ratio is attenuated and it quickly rises above the initial state, increasing the quarterly probability of a sovereign default close to half a percent above its steady state value. Due to the non-linear shape of the fiscal limit function, the increase in the probability of default implies a higher sensitivity of interest rates to fluctuations in economic fundamentals.

²²In the context of the third-order approximation to the policy functions, shown above, the difference between the two sets of impulse responses is almost entirely driven by the risk correction term $\sum_{i=0}^{\infty} \left(\frac{1}{2} \gamma \sigma_i^2 \sigma^2\right) \epsilon_{t-i}$. The effects by the second-order and third-order terms, describing actually realized fluctuations are negligible. See appendix A.3

²³For instance, Ramey (2011) estimates a fall of non-residential investment of 1% after a shock to defense spending, and a decline of residential investment of 3.5% in response to a government spending identified via professional forecast errors.

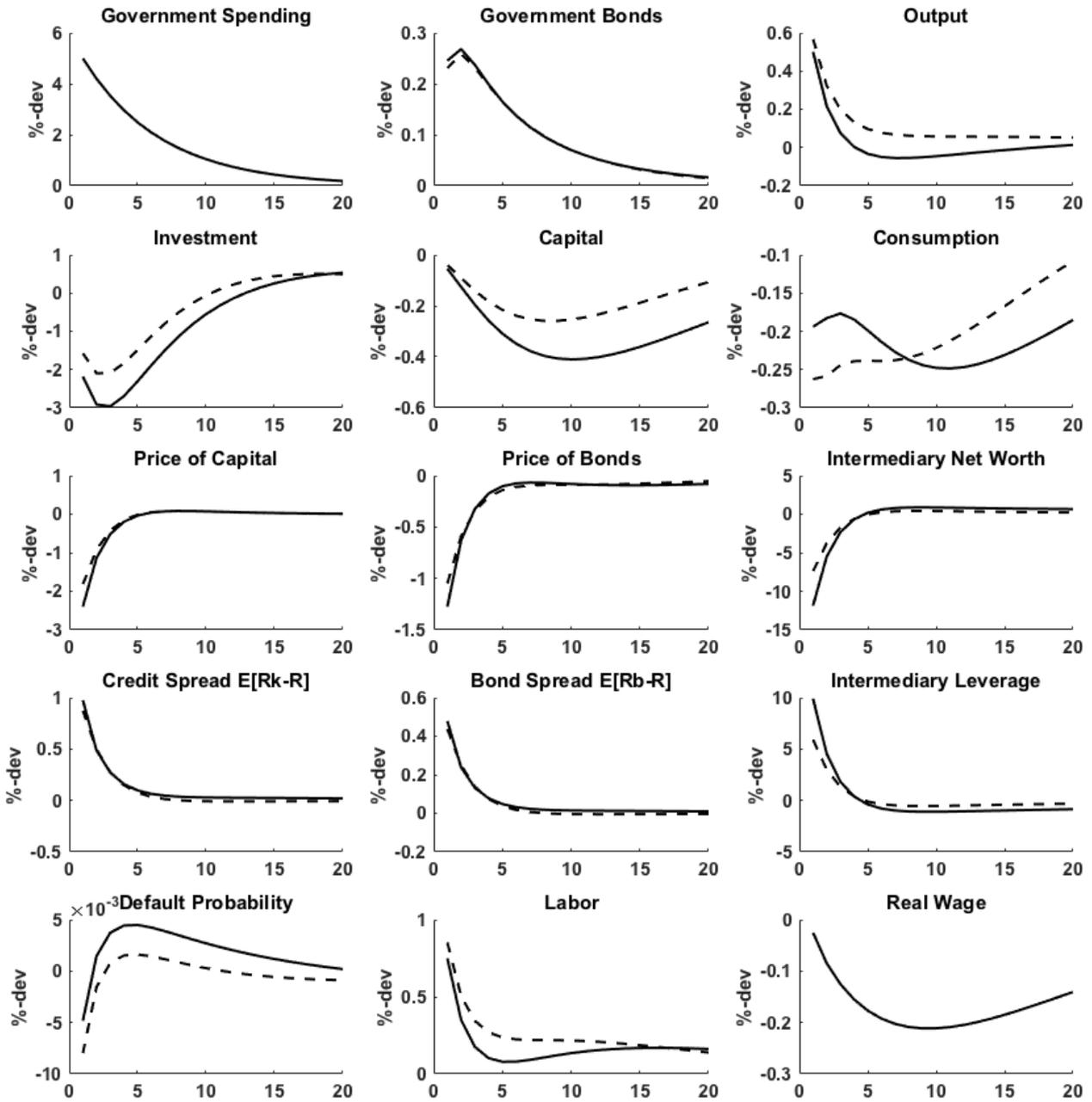


Figure 4: Dynamic consequences of a shock in government spending of 1 percent in the model with fiscal stress . The dashed lines depict the impulse responses obtained from a first-order approximation to equilibrium dynamics. The solid lines depicts the impulse responses obtained from a third-order approximation to equilibrium dynamics.

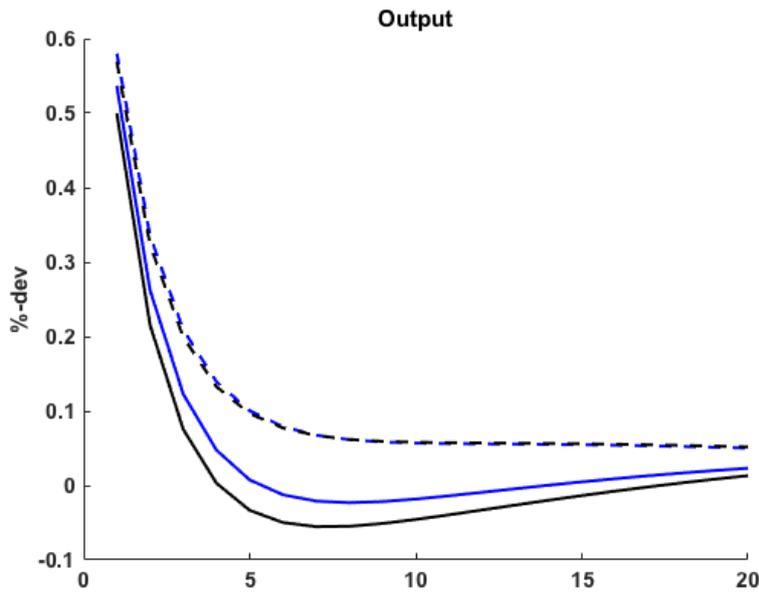


Figure 5: Output response to a shock in government spending of 1 percent obtained from a third-order approximation to equilibrium dynamics. The blue line depicts the impulse responses in the model without fiscal stress . The black line depicts the impulse responses obtained in the full model with fiscal stress .

While the current increase in the probability of default amplifies current losses, the outlook of a stronger response of the fiscal stress indicator and the economy to future shocks which move the debt-to-GDP ratio increases the motive of the banks to reduce their exposure to risky assets, contributing to the amplification of fire sales, the stronger contraction in investment, and the dampening of the output stimulus.

The contribution of the fiscal stress channel to the output response to a government spending shock is illustrated by figure (5). The black line represents the full model with fiscal stress and the blue line represents the case, in which the fiscal stress channel is shut off. The dashed lines represent the linear impulse responses (black - full model, blue - without fiscal stress). This figure shows that taking into account the effects of risk on the transmission of the shock alters the response in both cases with and without fiscal stress . The difference between the solid blue line and solid black line shows the impact of the fiscal stress channel on the reaction of aggregate output. While it is still relatively small, it is substantially larger than in the linear case, in which the two curves of the models with and without fiscal stress are virtually undistinguishable.

Table 2 shows that this is reflected by the resulting government spending multipliers. In the non-linear case without the fiscal stress channel, the impact multiplier is 0.54 and the cumulative multiplier is 0.16. Introducing fiscal stress lowers the impact multiplier to 0.50 and the cumulative multiplier to 0.06. Thus, whereas the introduction of the fiscal stress channel reduces the cumulative multiplier in the linearized economy by only 0.006, in the non-linear case, the differences of the cumulative multipliers with and without fiscal stress is roughly 15 times larger.

Table 2: Government Spending Multipliers

	Impact Multiplier	Cumulative Multiplier*
linear , no fiscal stress	0.581	0.368
linear , with fiscal stress	0.569	0.362
non-linear , no fiscal stress	0.537	0.155
non-linear , with fiscal stress	0.500	0.064

*time horizon of 20 quarters

Risk also influences the behavior of the other agents in the economy. For the risk-averse households it strengthens the motive of consumption smoothing and labor smoothing. Following the shock, households reduce their consumption by less, and, due to the smaller wealth effect, increase labor supply by less than in the linear case. Overall however, the steeper fall in investment outweighs the effect of the attenuation of the consumption response on output, enhancing the fall in output as well. A few periods after the shock, when the more pronounced decline in capital becomes effective over time, the response of aggregate consumption follows the fall in production and declines below its linear counterpart as well. [Born and Pfeifer \(2014\)](#) give an overview of further channels through which aggregate risk influences the behavior of households and firms in a model with sticky prices. However, as section 5 shows, in the model at hand the link between fiscal stress and the size of the multiplier virtually disappears, when bonds are not held by leverage constrained banks, I omit a detailed discussion of further channels.²⁴

This exercise illustrates two points. Firstly, the transmission of the government spending shock in general is altered, when risk about future shocks matters. This is mainly driven by the presence of financial frictions and fiscal stress. Secondly, the impact of the fiscal stress on the government spending is increased substantially by taking account of risk and precautionary motives of the agents, helping to generate a negative effect of fiscal stress on the government multiplier.

However, while the fiscal stress channel is larger than in the linear case, its effect is still too small to reflect the large effects that fiscal stress can have on the multiplier according to the empirical findings by [Perotti \(1999\)](#) or [Ilzetzki et al. \(2013\)](#). Given the benchmark calibration of the model, the latter result is not surprising. Since the benchmark calibration of the debt-to-GDP ratio and the parameters of the default function, which are taken from [Bi and Traum \(2012a\)](#), reflect a period in which Italian sovereign bond markets have been rather calm, the default function is relatively flat at the steady state, implying a low sensitivity of the default probability to changes in the debt-to-GDP ratio. Section 4.3 discusses under which conditions the link between fiscal stress and the government spending multiplier is strong.

5. When is the impact of fiscal stress on the multiplier strong?

The moderate role of the fiscal stress channel for the government spending multiplier in section 4 reflects that the parameter values for the benchmark calibration are obtained from a sample of Italian data from 1999.Q1-2010.Q3, a period which contains mostly years, in which sovereign yield spreads in Italy were low. This section assesses the role of the fiscal stress channel for the multiplier under conditions that capture some features of a sovereign debt crisis. In each

²⁴Results, for a version of the model in which government bonds are held by households are available upon request.

subsection, I adjust the benchmark calibration along one dimension. In particular, I focus on the debt-to-GDP ratio, η_2 , which governs the sensitivity of the fiscal stress indicator to the debt-to-GDP ratio, the leverage ratio of banks, the degree of aggregate risk, and κ_g , which determines, to what degree increases in government spending are mirrored by increases in taxes.

The figures in this section depict cumulative multipliers for the linear and the non-linear approximation (dashed and solid lines, respectively), both for the full model and the model without the fiscal stress channel (blue and black lines, respectively).

The main result for the multipliers obtained by a first-order approximation is that even for extreme calibrations the role of fiscal stress for the government spending multiplier is very small. This highlights the necessity for accounting for risk, when the aim is to generate a link between fiscal stress and the government spending multiplier. The results for the multipliers obtained by the non-linear approximation are discussed in their respective sections.

5.1 The role of fiscal stress when public debt is high

The benchmark calibration of the debt-to-GDP ratio in the deterministic steady state is 1.19, which is the average Italian debt-to-GDP in the sample by [Bi and Traum \(2012a\)](#). In recent years, Italy has experienced debt-to-GDP ratios of roughly 1.3. For the full model it holds that the higher the debt-to-GDP ratio the smaller is the cumulative multiplier, and the stronger is the fiscal stress channel. The solid black line in the upper panel of figure (6)) shows that this relationship is monotonous. This exercise points to the possibility that government spending multipliers have even been negative in Italy during recent crisis. As mentioned above, the possibility of negative fiscal multipliers has been discussed controversially following the proposition of the expansionary fiscal contraction hypothesis by [Giavazzi and Pagano \(1990\)](#).²⁵ In the context of the model at hand, negative government spending multipliers are possible, if public finances are sufficiently fragile, and financial intermediation is sufficiently impaired.

Changing the calibration of the debt-to-GDP ratio affects two mechanisms of the model, the portfolio shift of banks towards government bonds that is induced by an increase in government spending, and the effect of variations in the bond price in the necessity for banks to reduce the size of their balance sheets. Increases in the calibration of the debt-to-GDP increase the amount of public debt that banks hold in their portfolio and the strength of the crowding out of loans on the balance sheets through portfolio shifts. Thus for both models, with and without financial stress, the multiplier decreases for lower debt-to-GDP ratios.

In the model, in which the effect of fiscal stress on bond prices is eliminated (or very weak, as in the case of the linear approximation of the full model), the price of capital assets falls by more than the price of bonds after an expansionary government spending shock. This reflects that in the model without fiscal stress, the reason for the fall in bond prices is the fall in the price of capital assets, which is induced by the crowding out of investment. As the price of capital falls by more than the price of bonds, the fall in the average price of assets held by the banks is smaller, and therefore the need to delever is less severe, the larger the amounts of government bonds on the balance sheet relative to capital assets. Hence, a higher the debt-to-GDP ratio implies a less severe contraction in the banks' balance sheets after the shock. For debt-to-GDP ratios higher than 1.25, the latter effect dominates in the model without financial stress, and the multiplier increases with further increases in the debt-to-GDP ratio.

In the full model, in which government bond prices are affected by fiscal stress, they fall by more the higher the debt-to-GDP ratio, and the price effect decreases the multiplier for high

²⁵see, e.g., [Bertola and Drazen \(1993\)](#), [Alesina and Perotti \(1997\)](#), [Guajardo, Leigh, and Pescatori \(2011\)](#)

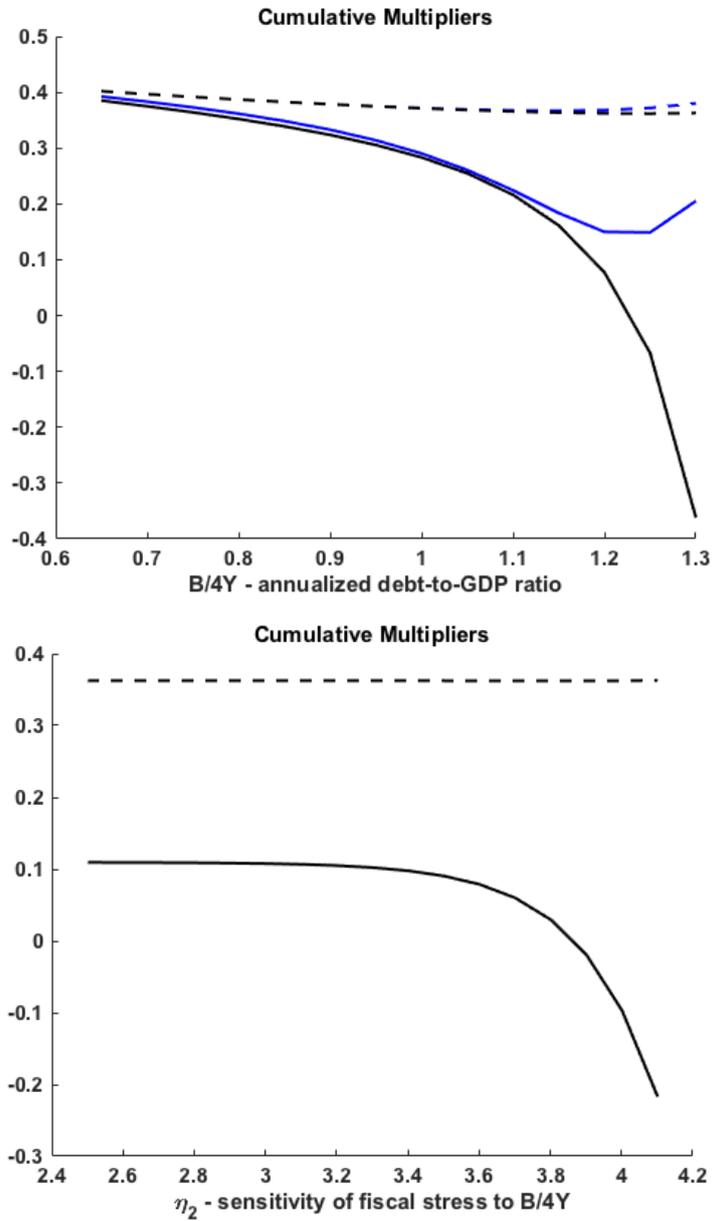


Figure 6: Cumulative government spending multipliers for a time-horizon of 20 periods. The x-axis shows the calibration of the parameters in the deterministic steady state. Blue dashed line - model without fiscal stress, linear approximation; Black dashed line - model with fiscal stress, linear approximation; Blue solid line - model without fiscal stress, non-linear approximation; Black solid line - model with fiscal stress, non-linear approximation

public debt. The multipliers of the model without fiscal stress and the full model diverge for large debt-to-GDP ratios. This illustrates that for large debt-to-GDP ratios, the effect of fiscal stress on the size of the government spending multiplier increases, allowing for a size of this effect that is closer to the what empirical evidence by [Perotti \(1999\)](#) and [Ilzetzki et al. \(2013\)](#) suggest.

5.2 The role of fiscal stress when it is very sensitive to the debt-to-GDP ratio

[Beirne and Fratzscher \(2013\)](#) suggest that during the recent debt crisis in the eurozone the sensitivity of default risk premia on government bonds to fundamentals became stronger for countries with weak public finances. The exercise in this section highlights that, if this sensitivity is increased in the model, fiscal stress can drastically lower cumulative multipliers, and, as in the previous subsection, even lead to negative multipliers. The more sensitive the fiscal stress indicator to variations in the debt-to-GDP ratio, the stronger is the fiscal stress channel, and the smaller is the cumulative multiplier. The lower panel of figure (6)) shows the sensitivity of the cumulative multiplier to η_2 , which governs the sensitivity of fiscal stress to the debt-to-GDP ratio. The benchmark calibration of this parameter value is 3.4015. The changes in the calibration of the debt-to-GDP ratio already entailed changes in the sensitivity of the fiscal stress indicator, as the latter is a convex function of the debt-to-GDP ratio. However, the sensitivity analysis with respect to η_2 isolates the price effects discussed in the previous section from the effects of variations in the steady state quantity of government bonds. As the model without fiscal stress does not feature a fiscal stress indicator, only the results for the full model are plotted. The higher η_2 the stronger are the degree of fiscal stress, bond prices, and the financial accelerator affected by variations in government spending. Thus, the contraction of the loan supply is stronger for higher values of η_2 , and the weaker is the output response induced by government spending shocks and the smaller the cumulative multiplier.

5.3 The role of fiscal stress when the degree of risk is high

The literature on stochastic volatility suggests macroeconomic and financial variables are higher, when the economy experiences a crisis.²⁶ In this subsection, I analyze how the degree of aggregate risk affect the government spending multiplier, and in particular, the role of the fiscal stress channel for the multiplier. Increasing the degree of risk does not affect the importance of the fiscal stress channel for the cumulative government spending multiplier. The upper panel of figure (7) depicts the relation between the degree of risk, scaled by factor k , and the multiplier in the model without and with fiscal stress. The benchmark calibration of k is 1. Concretely, in this exercise the standard deviations of all shocks except the government spending shock are premultiplied with k . Higher level of risk affect the multiplier mostly through a stronger effect of precautionary fire sales by banks, and a stronger financial accelerator. While figure (7) shows that for higher degrees of risk, the government multiplier decreases, the contribution of the fiscal stress channel to the size of the multiplier stays fairly stable.

²⁶see, e.g., [Bloom \(2009\)](#), [Basu and Bundick \(2015\)](#). [Bloom \(2009\)](#) calibrates the standard deviation of second moment shocks to TFP to a hundred percent of the standard deviation of the respective first moment shocks. [Basu and Bundick \(2015\)](#) calibrate the volatility of second moment shocks to a preference shock to be one sixths of the volatility of the respective level shock.

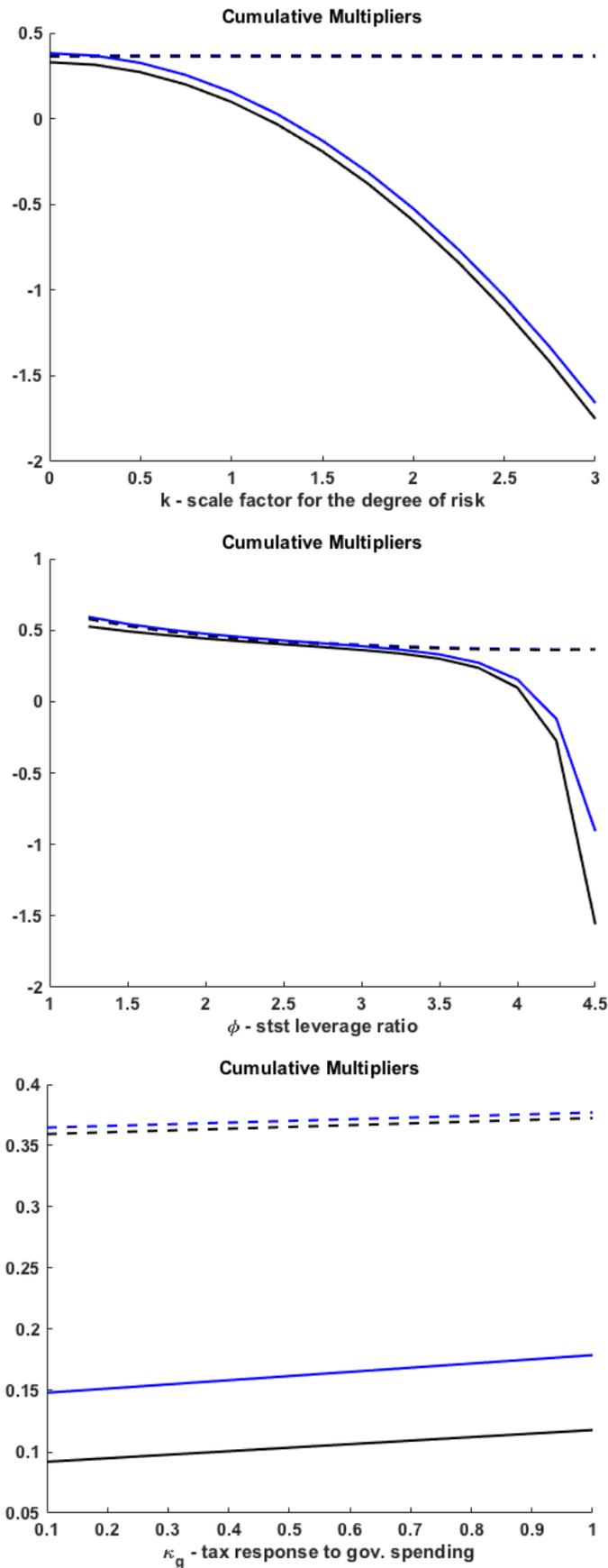


Figure 7: Cumulative government spending multipliers for a time-horizon of 20 periods. The x-axis shows the calibration of the parameters in the deterministic steady state. Blue dashed line - model without fiscal stress, linear approximation; Black dashed line - model with fiscal stress, linear approximation; Blue solid line - model without fiscal stress, non-linear approximation; Black solid line - model with fiscal stress, non-linear approximation

5.4 The role of fiscal stress when banks' leverage is high

Another feature of the recent debt crisis in the Eurozone were weakly capitalized banks. When banks are highly levered, fiscal stress has a stronger impact on the government spending multiplier. The middle panel of figure (7) depicts the relation between the leverage ratio in the deterministic steady state and the multiplier in the model without and with fiscal stress. The higher banks are levered up, the stronger is the need to delever in the face of adverse shocks to asset prices. As the fiscal stress channel amplifies the fall in the price of government bonds after an increase in government spending, it also amplifies the pressure on banks to delever after the shock. This stronger pressure on banks balance sheets becomes more notable, the higher the leverage ratio. Thus, the difference between the multipliers in the model with and without the fiscal stress channel increases for higher leverage ratios. Hence, the importance of fiscal stress for the government spending multiplier is likely to be higher, in crises such as the recent crisis in the Eurozone, in which banks were weakly capitalized, and is likely to decrease when banks' capital buffers are strengthened.

5.5 The role of fiscal stress when the share of debt-financing is high

Lastly, the government spending multiplier is lower for smaller shares of tax-financing of variations in government expenditures. The lower panel of figure (7) depicts the relation between the response coefficient for lump sum taxes to variations in government spending, κ_g , and the cumulative multiplier in the model without and with fiscal stress. For a value of 1, variations in government spendings are fully mimicked by changes in taxes. For low values, the share of debt-financing is higher. The higher the share of debt-financing, the more the amount of government bonds held by banks increases, and the stronger the crowding out effect of investment. Thus, the multiplier decreases in the share of debt-financing for both models. However, due to the low sensitivity of the bond price to changes in the debt-to-GDP ratio implied by the benchmark calibration, the difference between the multipliers in both models, and hence the importance of the fiscal stress channel for the government spending multiplier, is only weakly altered by variations in the share of debt-financing.

6. Conclusion

According to a growing empirical literature, the government spending multiplier is smaller for countries that experience fiscal stress, as opposed to countries, which have sound public finances. This paper develops a theoretical model with leverage constrained banks and sovereign default risk to explain the link between fiscal stress and the size of the government spending multiplier suggested by the empirical literature.

To the extent that a positive shock to government spending crowds out investment, it lowers the price of capital assets, which induces a downward pressure on the price of bonds as well. Additionally, the government spending shock increases the downward pressure on bond prices to the extent that it increases the debt-to-GDP ratio and raises the degree of fiscal stress. The fall of asset prices exerts an adverse effect on the balance sheets of banks, which are forced to sell assets, and contract the supply of credit to firms. As a consequence, aggregate investment falls and dampens the initial output stimulus of the government spending shock.

I show that whether the contribution of the fiscal stress channel to the response of output, and hence to the government spending multiplier, is quantitatively important depends on the order of approximation that is employed to solve the model. In the linearized model, the effect of the degree of fiscal stress on the government spending multiplier is negligible. In contrast to this,

fiscal stress can play a sizable role for the size of the multiplier, if the model is solved using a third-order approximation to equilibrium dynamics. The reason is that aggregate risk affects the transmission of the government spending shock. In particular, it amplifies the reduction of credit and investment, which can be explained by a precautionary motive of banks, who react more sensitive to variation in asset prices following the shock. As a result, the output stimulus induced by the shock, and the government spending multiplier is decreased by the presence of fiscal stress.

Considering the role of aggregate risk for the transmission of the government spending shock, is novel in the literature on government spending multipliers. Importantly, accounting for risk allows me to establish the empirically observed link between the degree of fiscal stress and the size of the government spending multiplier. I show that the fiscal stress channel for the multiplier becomes more important, the higher the initial degree of fiscal stress, in which government spending is increased, the higher the sensitivity of fiscal stress to fluctuations in the debt-to-GDP ratio, and the stronger the motive of banks to delever in the face of the shock. For a calibration, that captures some of the key features of the recent debt crisis in Italy, the cumulative government spending multiplier can even become negative.

While this paper offers a theoretical explanation for the empirical link between fiscal stress and the size of the multiplier, it does not attempt a full-fledged analysis of a sovereign risk crisis. Instead it focusses on a key mechanism, and highlights the role of risk for the fiscal stress channel. Additionally, episodes with increased sovereign default risk can be associated with steep recessions, or liquidity problems of the agents in the economy, that may affect the government spending multiplier. Furthermore, aspects such as distortionary taxation, the political economy, unconventional monetary policy, and open economy considerations are omitted from the analysis to focus on the contribution of the fiscal stress channel to the government spending multiplier. The many possible extensions of the analysis yield an interesting field of future research.

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A Appendix

A.1 Bank's optimization problem

The bank maximizes its value function subject to a balance sheet constraint and an incentive constraint:

$$V_{jt} = \max_{\{K_{jt}\}, \{B_{jt}\}, \{D_{jt}\}} E_t \Lambda_{t,t+1} [(1 - \theta) N_{jt} + \theta V_{j,t+1}]$$

$$s.t. \quad Q_t K_{jt} + Q_t^b B_{jt} = D_{jt} + N_{jt}$$

$$V_{jt} \geq \lambda Q_t K_{jt} + \lambda_b Q_t^b B_{jt}$$

In the following, I make the assumption that the incentive constraint is always binding. Additionally, the law of motion of net worth is assumed to be:

$$N_{jt} = (R_{kt} - R_{t-1}) Q_{t-1} K_{j,t-1} + (R_{bt} - R_{t-1}) Q_{t-1}^b B_{j,t-1} + R_t N_{j,t-1}.$$

Guess that the value function is linear in loans, government bonds and net worth

$$V_{jt} = v_{kjt} Q_t K_{jt} + v_{bjt} Q_t^b B_{jt} + v_{njt} N_{jt}.$$

The Lagrangian function for the optimization problem of the bank reads:

$$\mathcal{L} = (1 + \mu_{jt})(v_{kjt} Q_t K_{jt} + v_{bjt} Q_t^b B_{jt} + v_{njt} N_{jt}) - \mu_{jt}(\lambda Q_t K_{jt} + \lambda_b Q_t^b B_{jt})$$

Hence, the first order conditions for loans, bonds, and the Lagrangian multiplier, μ_t , are:

$$v_{kjt} = \lambda \frac{\mu_{jt}}{1 + \mu_{jt}}$$

$$v_{bjt} = \lambda_b \frac{\mu_{jt}}{1 + \mu_{jt}}$$

$$v_{kjt} Q_t K_{jt} + v_{bjt} Q_t^b B_{jt} + v_{njt} N_{jt} = \lambda Q_t K_{jt} + \lambda_b Q_t^b B_{jt}$$

The supply of loans can be obtained by rearranging the incentive constraint:

$$Q_t K_{jt} = \frac{v_{bjt} - \lambda_b}{\lambda - v_{kjt}} Q_t^b B_{jt} + \frac{v_{njt}}{\lambda - v_{kjt}} N_{jt}$$

The Value function can be written solely as a function of N_{jt} . by substituting out private assets using the foregoing equation:

$$V_{jt} = v_{kjt} Q_t K_{jt} + v_{bjt} Q_t^b B_{jt} + v_{njt} N_{jt}$$

$$\Leftrightarrow V_{jt} = \left[v_{kjt} \frac{v_{bjt} - \lambda_b}{\lambda - v_{kjt}} + v_{bjt} \right] Q_t^b B_{jt} + \left[v_{kt} \frac{v_{njt}}{\lambda - v_{kjt}} + v_{njt} \right] N_{jt}$$

$$\Leftrightarrow V_{jt} = \left[v_{kt} \frac{v_{njt}}{\lambda - v_{kjt}} + v_{njt} \right] N_{jt}$$

$$\Leftrightarrow V_{jt} = \left[\frac{\lambda v_{njt}}{\lambda - v_{kjt}} \right] N_{jt}$$

$$\Leftrightarrow V_{jt} = \left[\frac{\lambda v_{njt}}{\lambda - \lambda \frac{\mu_{jt}}{1 + \mu_{jt}}} \right] N_{jt}$$

$$\Leftrightarrow V_{jt} = (v_{njt}(1 + \mu_{jt})) N_{jt}$$

Defining: $\Omega_{j,t} \equiv \Lambda_{t-1,t}((1-\theta) + \theta(1 + \mu_{jt})v_{njt})$, plugging this expression of the value function into the Bellman equation, and using the law of motion of net worth, yields:

$$\begin{aligned} V_{jt} &= v_{kjt} Q_t K_{jt} + v_{bjt} Q_t^b B_{jt} + v_{njt} N_{jt} \\ &= \beta E_t \Lambda_{t,t+1} [(1-\theta) N_{jt} + \theta V_{j,t+1}] \\ &= \beta E_t \Omega_{t+1} ((R_{k,t+1} - R_t) Q_t K_{j,t} + (R_{b,t+1} - R_t) Q_t^b B_{j,t} + R_t N_{j,t}). \end{aligned}$$

One can then solve for the coefficients of the value function:

$$\begin{aligned} v_{kjt} &= \beta E_t \Omega_{jt+1} (R_{k,t+1} - R_t), \\ v_{bjt} &= \beta E_t \Omega_{jt+1} (R_{b,t+1} - R_t), \\ v_{dj t} &= \beta E_t \Omega_{jt+1} R_t. \end{aligned}$$

For aggregation, I assume an equilibrium in which all banks are symmetric (i.e., $\forall j : v_{kjt} = v_{kt}, v_{bjt} = v_{bt}, v_{njt} = v_{nt}, \Omega_{jt+1} = \Omega_{t+1}$). The leverage ratio, ϕ_t , is obtained by defining the ratio of bonds over loans in the portfolio of the bank, $\zeta_t = \frac{Q_t^b B_t}{Q_t K_t}$ and starting from the incentive constraint:

$$\begin{aligned} (\lambda - v_{kt}) Q_t K_t + (\lambda_b - v_{bt}) Q_t^b B_t &= v_{nt} N_t \\ \Rightarrow (\lambda - v_{kt}) Q_t K_t + \left(\frac{\lambda v_{bt} - v_{kt} v_{bt}}{v_{kt}} \right) Q_t^b B_t &= v_{nt} N_t \\ \Rightarrow (\lambda - v_{kt}) Q_t K_t + \left((\lambda - v_{kt}) \frac{\lambda_b}{\lambda} \right) Q_t^b B_t &= v_{nt} N_t \\ \Rightarrow (\lambda - v_{kt}) Q_t K_t + (\lambda - v_{kt}) \frac{\lambda_b}{\lambda} \zeta_t Q_t K_t &= v_{nt} N_t \\ \Rightarrow Q_t K_t &= \frac{v_{nt}}{(\lambda - v_{kt})(1 + \frac{\lambda_b}{\lambda} \zeta_t)} N_t \\ \Rightarrow \frac{Q_t K_t + Q_t^b B_t}{N_t} &= \frac{v_{nt}(1 + \zeta_t)}{(\lambda - v_{kt})(1 + \frac{\lambda_b}{\lambda} \zeta_t)} \equiv \phi_t \end{aligned}$$

A.2 Equilibrium equations

Households

$$W_t = \frac{\chi L_t^\phi}{U_{c,t}} \quad (\text{A-1})$$

$$U_{c,t} = (C_t - hC_{t-1})^{-1} - \beta h E_t (C_{t+1} - hC_t)^{-1} \quad (\text{A-2})$$

$$1 = E_t \beta \Lambda_{t,t+1} R_t \quad (\text{A-3})$$

$$\Lambda_{t,t+1} = \frac{U_{c,t+1}}{U_{c,t}} \quad (\text{A-4})$$

Intermediate Good Producers

$$Y_{mt} = A_t (\xi_t U_t K_{t-1})^\alpha L_t^{1-\alpha} \quad (\text{A-5})$$

$$\delta(U_t) = \delta_c + \frac{b}{1+\zeta} * U_t^{1+\zeta} \quad (\text{A-6})$$

$$R_{k,t+1} = \frac{P_{m,t+1} \alpha \frac{Y_{m,t+1}}{K_t} + (1 - \delta(U_{t+1})) Q_{t+1} \xi_{t+1}}{Q_t} \quad (\text{A-7})$$

$$W_t = P_{mt} (1 - \alpha) \frac{Y_{mt}}{L_t} \quad (\text{A-8})$$

$$\delta'(U_t) \xi_t Q_t K_{t-1} = P_{mt} \alpha \frac{Y_{mt}}{U_t} \quad (\text{A-9})$$

Capital Good Producers

$$K_t = (1 - \delta(U_t)) \xi_t K_{t-1} + I_t \quad (\text{A-10})$$

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}} f'\left(\frac{I_t}{I_{t-1}}\right) - E_t \beta \Lambda_{t,t+1} \left(\frac{I_t}{I_{t-1}}\right)^2 f'\left(\frac{I_t}{I_{t-1}}\right) \quad (\text{A-11})$$

$$f\left(\frac{I_t}{I_{t-1}}\right) = \frac{\eta_i}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2 \quad (\text{A-12})$$

Retailers

$$\Pi_t^* = \frac{\epsilon}{\epsilon - 1} \frac{F_t}{Z_t} \Pi_t \quad (\text{A-13})$$

$$F_t = Y_t P_{mt} + \beta \gamma \Lambda_{t,t+1} \Pi_{t+1}^\epsilon \Pi_t^{-\gamma p \epsilon} F_{t+1} \quad (\text{A-14})$$

$$Z_t = Y_t + \beta \gamma \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon-1} \Pi_t^{-\gamma p (\epsilon-1)} Z_{t+1} \quad (\text{A-15})$$

$$Y_{mt} = \Delta_{p,t} Y_t \quad (\text{A-16})$$

$$\Delta_{p,t} = \gamma \Delta_{p,t-1} \Pi_t^\epsilon \Pi_{t-1}^{-\gamma p \epsilon} + (1 - \gamma) \left(\frac{1 - \gamma \Pi_t^{\epsilon-1} \Pi_{t-1}^{-\gamma p (\epsilon-1)}}{1 - \gamma} \right)^{\frac{\epsilon}{\epsilon-1}} \quad (\text{A-17})$$

$$X_t = \frac{1}{P_{mt}} \quad (\text{A-18})$$

$$\Pi_t^{1-\epsilon} = (1 - \gamma) (\Pi_t^*)^{1-\epsilon} + \gamma \Pi_{t-1}^{\gamma p (1-\epsilon)}, \quad (\text{A-19})$$

Banks

$$v_{kjt} = \lambda \frac{\mu_{jt}}{1 + \mu_{jt}} \quad (\text{A-20})$$

$$v_{bjt} = \lambda_b \frac{\mu_{jt}}{1 + \mu_{jt}} \quad (\text{A-21})$$

$$Q_t K_{jt} = \frac{v_{bjt} - \lambda_b}{\lambda - v_{kjt}} Q_t^b B_{jt} + \frac{v_{njt}}{\lambda - v_{kjt}} N_{jt} \quad (\text{A-22})$$

$$\Omega_{j,t} \equiv \Lambda_{t-1,t} ((1 - \theta) + \theta (1 + \mu_{jt}) v_{njt}) \quad (\text{A-23})$$

$$v_{kjt} = \beta E_t \Omega_{j,t+1} (R_{k,t+1} - R_t) \quad (\text{A-24})$$

$$v_{bjt} = \beta E_t \Omega_{j,t+1} (R_{b,t+1} - R_t) \quad (\text{A-25})$$

$$v_{njt} = \beta E_t \Omega_{j,t+1} R_t \quad (\text{A-26})$$

$$\phi_t \equiv \frac{v_{nt} (1 + \zeta_t)}{(\lambda - v_{kt}) (1 + \frac{\lambda_b}{\lambda} \zeta_t)} = \frac{Q_t K_t + Q_t^b B_t}{N_t} \quad (\text{A-27})$$

$$Q_t K_t + Q_t^b B_t = D_t + N_t \quad (\text{A-28})$$

$$\varsigma_t = \frac{Q_t^b B_t}{Q_t K_t} \quad (\text{A-29})$$

$$N_{ot} = \theta \left[R_{kt} Q_{t-1} K_{t-1} + R_{bt} Q_{t-1}^b B_{t-1} - R_{t-1} D_{t-1} \right] \quad (\text{A-30})$$

$$N_{nt} = \omega \left[Q_{t-1} K_{t-1} + Q_{t-1}^b B_{t-1} \right] \quad (\text{A-31})$$

$$N_t = N_{ot} + N_{nt} \quad (\text{A-32})$$

$$prem_t = \frac{E_t R_{kt+1}}{R_t} \quad (\text{A-33})$$

Fiscal Policy

$$G_t = G e^{g_t} \quad (\text{A-34})$$

$$T_t = T + \kappa_b (B_{t-1} - B) + \kappa_g (G_t - G) \quad (\text{A-35})$$

$$R_{b,t} = (1 - \Delta_t^d * D) \left[\frac{r_c + \rho_c Q_t^b}{Q_{t-1}^b} \right] \quad (\text{A-36})$$

$$G_t + Q_t^b B_t = R_{bt} Q_{t-1}^b B_{t-1} + T_t \quad (\text{A-37})$$

$$\Delta_t^d = \frac{\exp\left(\eta_1 + \eta_2 \frac{B_t}{4Y_t}\right)}{1 + \exp\left(\eta_1 + \eta_2 \frac{B_t}{4Y_t}\right)} \quad (\text{A-38})$$

$$prem_t^b = \frac{E_t R_{bt+1}}{R_t} \quad (\text{A-39})$$

Monetary Policy and Good Market Clearing

$$i_t = \rho_i i_{t-1} + (1 - \rho)(i + \kappa_\pi \pi_t + \kappa_y \hat{m}c_t) + \epsilon_t^i \quad (\text{A-40})$$

$$1 + i_t = R_t \frac{E_t P_{t+1}}{P_t} \quad (\text{A-41})$$

$$Y_t = C_t + I_t + f\left(\frac{I_t}{I_{t-1}}\right) I_t + G_t \quad (\text{A-42})$$

Shock Processes

$$g_t = \rho_g g_{t-1} + \epsilon_t^g \quad (\text{A-43})$$

$$a_t = \rho_a a_{t-1} + \epsilon_t^a \quad (\text{A-44})$$

$$\xi_t = \rho_\xi \xi_{t-1} + \epsilon_t^\xi \quad (\text{A-45})$$

A.3 Decomposition of the output response to the shock (benchmark case)

This section highlights, that the time-varying risk adjustment is indeed the main reason for the difference between the third-order accurate impulse response of output to the government spending shock and the first-order accurate impulse response to the government spending shock. Figure (8) illustrates the contributions of the different components of the policy function to the overall response of output to the shock. For convenience, equation (49) is restated below

$$y_t = \bar{y} + \frac{1}{2} y_{\sigma^2} \sigma^2 + \sum_{i=0}^{\infty} \left(y_i + \frac{1}{2} y_{\sigma_i^2} \sigma^2 \right) \epsilon_{t-i} + \frac{1}{2} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} y_{i,j} (\epsilon_{t-i} \otimes \epsilon_{t-j}) + \frac{1}{6} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} y_{i,j,k} (\epsilon_{t-i} \otimes \epsilon_{t-j} \otimes \epsilon_{t-k}),$$

where \bar{y} denotes the vector of deterministic steady state values of the respective endogenous variables, and the matrices of partial derivatives, $y_i, y_{i,j}, y_{i,j,k}, y_{\sigma^2}$ and $y_{\sigma_i^2}$, are evaluated at the deterministic steady state. σ is a scale factor for the degree of risk in the model, and $\epsilon_{t-i}, \epsilon_{t-j}$, and ϵ_{t-k} are vectors of past realizations of shocks.

The left panel of the middle row of figure (8) depicts the first-order accurate response to the realized government spending shock, which is captured by the term $\sum_{i=0}^{\infty} y_i \epsilon_{t-i}$ in the policy function. Note, that the contribution of the first-order component is at the scale of 10^{-3} . The lower left panel depicts the contribution of the time-varying risk adjustment term, $\frac{1}{2} \sum_{i=0}^{\infty} y_{\sigma_i^2} \sigma^2 \epsilon_{t-i}$.

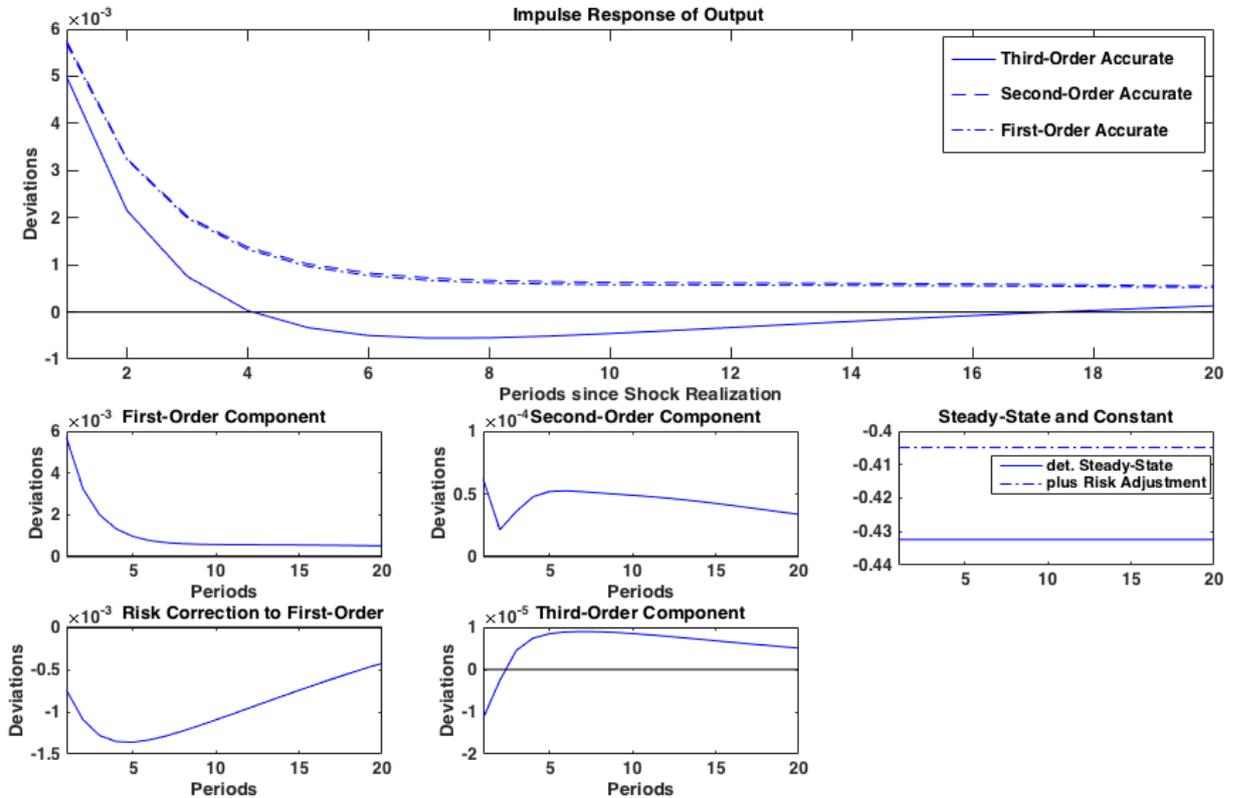


Figure 8: Components of the output response to a shock in government spending of 1 percent of output.

Again, the contribution of this term the third-order accurate impulse response is of the order 10^{-3} . In contrast to these terms, for the same impulse response, the contributions of the second-order component, $\frac{1}{2} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \mathcal{Y}_{i,j}(\epsilon_{t-i} \otimes \epsilon_{t-j})$ (depicted in the center panel), and the third-order component, $\frac{1}{6} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \mathcal{Y}_{i,j,k}(\epsilon_{t-i} \otimes \epsilon_{t-j} \otimes \epsilon_{t-k})$ (depicted in the lower right panel), are of smaller magnitudes (10^{-4} and 10^{-5} , respectively). Thus the first-order component and the risk-adjustment are the dominating terms for the third-order accurate impulse response.

The solid line in the upper panel of figure (8) shows the overall impulse response of output to the government spending shock that is obtained with a third-order approximation. Compared to the first-order accurate and the second-order accurate impulse response it displays a weaker impact of the shock on output. The lower left panel of figure (8) shows that this difference is due to the negative contribution of the risk correction to first-order to the impact of the government spending shock on output. This reflects the fact that in the face of the shock banks reduce their exposure to the risky assets by more than in the linear setting, decreasing the credit supply, investment and hence, output.

As the state-dependence of impulse responses, and the asymmetry between responses to positive and negative shocks hinge on the second-order terms and the third-order terms associated with realized shocks, they are very small, and I can abstract from them in my analysis.