The recent sovereign debt crisis in the Eurozone was characterized by a monetary policy, which has been constrained by the zero lower bound (ZLB) on nominal interest rates, and several countries, which faced high risk spreads on their sovereign bonds. How is the government spending multiplier affected by such an economic environment? While prominent results in the academic literature point to high government spending multipliers at the ZLB, higher public indebtedness is often associated with small government spending multipliers. I develop a DSGE model with leverage constrained banks that captures both features of this economic environment, the ZLB and fiscal stress. In this model, I analyze the effects of government spending shocks. I find that not only are multipliers large at the ZLB, the presence of fiscal stress can even increase their size. For longer durations of the ZLB, multipliers in this model can be considerably larger than one.

Keywords: Government spending multiplier; Fiscal stress; Zero lower bound; Financial frictions

JEL Classification: E32, E 44, E62, H30, H60
1. Introduction

Following the global financial crisis, which started in 2007, several European countries have experienced deep and persistent recessions as well as spiking sovereign yield spreads. At the same time, the ECB eased its monetary policy until the main refinancing rate hit the zero lower bound (ZLB). In this situation, these countries faced two challenges simultaneously: restoring the sustainability of their public finances while limiting the deep recession. With monetary policy being constrained by the ZLB, fiscal policy came into the focus of the public policy debate. While countries under fiscal stress ran policies of fiscal austerity, which included cuts to government spending, to stabilize their public debt, it is far from clear to what extent the government spending cuts contributed to the contraction of aggregate output in these countries. On the one hand, prominent result point toward large government spending multipliers at the ZLB,\(^1\) on the other hand, higher public indebtedness is often associated with small government spending multipliers.\(^2\) This paper investigates on the size of the government spending multiplier in the context of a DSGE model with leverage constrained banks that captures both features of this economic environment, the ZLB and fiscal stress.

For the analysis at hand, I set up a medium scale DSGE model with financial frictions. A key feature of this model are endogenously leverage constrained banks as in Gertler and Karadi (2011), which, in addition to private capital assets, hold government bonds that are subject to default risk. As stressed by van der Kwaak and van Wijnbergen (2014), the exposure of banks in the Eurozone to domestic sovereign bonds is substantial. Thus, fluctuations in the degree of fiscal stress are likely to affect the conditions of the banks’ asset side, and therefore their ability to supply loans to firms. Indeed, as highlighted by Corsetti et al. (2013), sovereign risk spreads and private credit spreads have been highly correlated in those countries in the Eurozone, which experienced increasing degrees of fiscal stress. Introducing the link between fragile banks and risky government bonds into the model allows me to capture that variations in government spending, to the extent that they affect the degree of fiscal stress, can have an immediate impact on the credit supply, and therefore on investment in the economy. Fiscal stress is captured in the model by the probability of a sovereign default modeled in the form of a fiscal limit function as discussed by Leeper and Walker (2011) and used in similar variations by other authors.\(^3\) This function maps the debt-to GDP ratio into the probability of a sovereign default. Following Gertler and Karadi (2011), I simulate a negative capital quality shock to trigger a financial crisis scenario in which the economy is forced to the ZLB on nominal interest rates, and compare government spending multipliers in the state, in which the ZLB constraint is slack, to multipliers in the state, in which the ZLB constraint is binding. To solve the model with the occasionally binding constraint, I employ the piece-wise linear approach by Guerrieri and Iacoviello (2015) and their software toolkit OccBin.

In the baseline scenario of the model, the cumulative government spending multiplier over 20 quarters is roughly 0.39 when the ZLB constraint is not binding. When the economy is not at the ZLB, I find that an increase in government spending raises the debt-to-GDP ratio and thereby the degree of fiscal stress, lowering the price of government bonds and capital assets, and raising the loan rate. Hence, the increase in the degree of fiscal stress contributes to a crowding-out of investment and lowers the government spending multiplier. However, the log-linear dynamics of the model imply that the size of the effect of fiscal stress on the government spending multiplier

\(^1\)see, e.g., Christiano, Eichenbaum, and Rebelo (2011) and Eggertson and Krugman (2012)

\(^2\)see, e.g., Alesina and Perotti (1997), Perotti (1999), Corsetti, Kuester, Meier, and Müller (2013)

\(^3\)Examples include: Bi and Traum (2012a), Bi and Traum (2012b), Corsetti et al. (2013), van der Kwaak and van Wijnbergen (2013), van der Kwaak and van Wijnbergen (2015), Bi, Leeper, and Leith (2014).
is negligible, when the ZLB constraint is slack.

When the ZLB constraint is binding, however, the picture changes drastically. In this state, the output response to a positive shock to government spending is far larger, and an expansionary fiscal policy now manages to improve the financial condition of banks and to crowd in investment. The longer the duration of the ZLB episode, the larger the government spending multiplier. Multipliers in this model can become larger than two. Also the effect of the fiscal stress channel on the multiplier is reversed, when monetary policy is constrained and contributes positively to the multiplier.

As emphasized by Eggertson (2011) and Christiano et al. (2011), the key to the different sizes of multipliers at the ZLB is the reversed effect of government spending shocks on the real interest rate. An expansion of government spending raises aggregate demand and triggers an increase in output and inflation. In normal times, when the central bank follows the Taylor principle for nominal interest rates, this causes the real interest rate to go up as well.

At the ZLB the effect on the real interest rate changes. In the scenario at hand, the capital quality shock that triggers the crisis, causes inflation to drop. With monetary policy constrained and nominal interest rates stuck at zero, the expectations of a persistent deflation translates into a rising real interest rate. An increase in government spending that raises aggregate demand and exerts inflationary pressure therefore leads to a decrease in the real interest rate. Thus, expansionary fiscal policy makes investment more attractive, when the economy is at the ZLB.

The presence of the fiscal stress channel in my model does not undo this intuition, but rather strengthens it. Despite the growing public debt after a positive government spending shock, the large output response lowers the debt-to-GDP ratio and the degree of fiscal stress. Particularly, for higher levels of public indebtedness, for which the default probability is more sensitive to movements in underlying fundamentals, the falling degree of fiscal stress raises bond prices and strengthens the balance sheet of banks and their ability to supply credit. Hence, the fiscal stress channel further amplifies the investment boom, and contributes to a large multiplier at the ZLB.

Empirical evidence on the government spending multipliers at the ZLB is scant as ZLB episodes have been rare in industrialized countries. Miyamoto, Nguyen, and Sergeyev (2016) analyze the case of Japan in which the policy rate has been close to or at the ZLB since the mid 1990’s. Using local projection methods, they estimate the output multiplier in Japan to be 1.5 at the ZLB and 0.6 when the ZLB is not binding. Their results are roughly in line with the multipliers, I obtain in my paper. Empirical papers, that study cases outside of Japan, have to rely either on historical data or on the few years of the Great Recession. Ramey and Zubairy (2014) analyze US data reaching back to 1889, including two ZLB episodes. While the government spending multiplier is not significantly different from normal times in an estimation over the entire sample, it becomes significantly larger, when the ZLB episode around World War II is excluded and only the Great Recession remains. For the interwar period in the UK, in which the interest rates were at the lower bound, Crafts and Mills (2013) find government spending multipliers below one for different shock identification schemes.

In the theoretical literature on the government spending multiplier, a large multiplier at the ZLB is a recurring result. This result is in almost all cases explained by the non-standard reaction of the real interest rate to government spending shocks explained above. In his

\[ \text{\footnotesize{see, e.g., Woodford (2010), Christiano et al. (2011), Eggertson (2011), Eggertson and Krugman (2012), Aloui and Eyquem (2016). As a caveat, however, one should point out that most of these results are obtained relying on log-linear equilibrium conditions. Recently, Boneva, Braun, and Waki (2016) showed that in the context of a small New-Keynesian model, which is solved non-linearly, the size of the multiplier is very sensitive to the calibration of the model and can take very different values for plausible calibrations.}} \]
careful analysis of the effects of different fiscal policy shocks at the ZLB, Eggertson (2011) obtains a government spending multiplier of 2.3. Christiano et al. (2011) find roughly the same value.

On the other hand, empirical evidence on the role of fiscal stress for the size of the government spending multiplier generally suggests that higher public debt, higher deficits or higher sovereign yield spreads are associated with small government spending multipliers. Some results even point to negative multipliers for countries, which are highly indebted. Ilzetzki, Mendoza, and Végh (2013) estimate the long run multiplier to be negative three. Theoretical models that investigate the role of fiscal stress for the government spending multipliers offer different explanations for the observation of 'expansionary fiscal contractions'. Bertola and Drazen (1993) argue that, if a fiscal contraction is associated with a shift to a sustainable path of government debt, it can spur expectations of future economic growth and stimulate consumption by households.

The papers closest to the one presented here are Corsetti et al. (2013) and Aloui and Eyquem (2016). Both analyze the multiplier in a model that takes account of both, fiscal stress and the ZLB. In the context of a small New-Keynesian model with banks à la Cúrdia and Woodford (2011) and a fiscal limit function, Corsetti et al. (2013) find that multipliers can actually be smaller at the ZLB, as in this situation monetary policy cannot counter the recessionary effects of sharply increasing sovereign yield spreads. However, more recently, Aloui and Eyquem (2016) extend the model by Corsetti et al. (2013) with capital formation, investment adjustment costs and distortionary labor taxes, and show that in the extended model, the typical result that multipliers are increased by the ZLB is restored. While my framework differs along various dimensions from theirs, I confirm the finding by Aloui and Eyquem (2016). In my model as well, the ZLB dominates the fiscal stress channel as a determinant of the size of the government spending multiplier leading to a multiplier’s size that is in the range of values typically found in the related literature.

The remainder of the paper is structured as follows: Section two gives a brief overview of the model. Section three discusses the calibration and the solution method. The fourth section introduces the crisis experiment, analyzes the dynamic consequences of government spending shocks at the ZLB, and discusses government spending multipliers for different lengths of the ZLB period and different levels of indebtedness. Section five concludes.

The environment

The model builds on the framework used by Gertler and Karadi (2011), and adds an extra twist to make it suitable for the analysis of the link between fiscal stress and the government spending multiplier. In particular, in addition to capital assets, banks hold government bonds as a second asset on their balance sheets, and the default probability of government bonds is tied to the debt-to-GDP ratio. Time is discrete, and one period in the model represents one quarter. The model features households, banks, intermediate good producers, capital good producers, retailers, a fiscal and a monetary authority. Figure (1) provides an illustration of the model structure. Households consume, supply labor, and save in the form of bank deposits. The firm sector consists of three types of firms. Intermediate good producers employ labor and capital to produce their goods. Each period, after producing their output, they sell their used capital stock to the capital goods producers. The latter repair it, and invest in new capital. At the end of the period they re-sell

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the capital to the intermediate good producers, which use it for production in the next period. The intermediate good producers finance their purchases of capital with loans from the banks. Intermediate goods are purchased by retailers, which repackage them, and sell them with a markup as final goods to households, the capital producers, and to the government. Banks hold loans and government bonds on the asset side of their balance sheets. On the liability side are deposits and the banks net worth. The government consumes final goods, collects taxes, and issues government bonds, which are subject to default risk. Monetary policy takes the form of a Taylor rule. The model includes habit formation in consumption, convex investment adjustment costs, sticky prices, and price indexation to enhance the empirical plausibility of the model dynamics, and to facilitate the comparability of my results with the results by other authors which have used this framework.

**Households**

There is a continuum of households with a unit mass. As in Gertler and Karadi (2011) a constant fraction \( f \) of each household’s members works as banker, whereas the other fraction \((1 – f)\) consists of workers who supply labor to the intermediate good producers. While workers receive their wage income every period, bankers reinvest their gains in asset holdings of the bank over several periods, and contribute to the households income only when exiting the banking sector, bringing home the accumulated profits. To ensure that both fractions of the household face the same consumption stream, perfect consumption insurance within the household is assumed. Households’ expected lifetime utility is as follows

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln(C_t - hC_{t-1}) - \frac{X}{1 + \phi} L_{t+1} - \phi \right],
\]
where $C_t$ is consumption and $L_t$ is labor that the workers supply to intermediate good producers. $\beta$ is the discount factor, $h$ is the parameter of the habit formation, $\phi$ is the inverse of the Frisch elasticity, and $\chi$ scales the weight of the disutility from labor in the preferences. Households can save via a one period bank deposit, which earns the riskless interest rate, $R_t$. The income stream of the household is thus composed of the wage income $W_t L_t$, banker’s profits $\Upsilon^b_t$, firm profits, $\Upsilon^f_t$ net the payment of lump sum taxes $T_t$. It uses this income to purchase consumption goods or to renew its deposits. The budget constraint thus reads

$$C_t + D_t = W_t L_t + R_{t-1} D_{t-1} + \Upsilon^b_t + \Upsilon^f_t - T_t.$$  

Maximizing life-time utility with respect to consumption, labor and deposit holdings subject to the sequence of budget constraints yields the first order conditions of the household

$$W_t = \chi L_t^{\phi}, $$

$$U_{c,t} = (C_t - h C_{t-1})^{-1} - \beta h E_t (C_{t+1} - h C_t)^{-1}, $$

$$1 = E_t \beta \Lambda_{t,t+1} R_t,$$

with

$$\Lambda_{t,t+1} = \frac{U_{c,t+1}}{U_{c,t}}.$$  

Firm sectors

The model contains three types of firms. Intermediate goods are produced by perfectly competitive firms, which use capital and labor as inputs for production. Monopolistically competitive retailers buy a continuum of intermediate goods, and assemble them into a final good. Nominal frictions as in Calvo (1983) make the retailers optimization problem dynamic. Additionally, a capital producing sector buys up capital from the intermediate good producer, repairs it, and builds new capital, which it sells to the intermediate good sector again. Investment in new capital is subject to investment adjustment costs.

Intermediate good producers

In this setup dynamic pricing and investment decisions are carried out by retailers and capital good producers, respectively. Thus, the optimization of the intermediate good producers can be reduced to a sequence of static problems. Their production function takes a standard Cobb Douglas form, given by

$$Y_{mt} = A_t (\xi_t K_{t-1})^\alpha L^1 - \alpha,$$

where $0 < \alpha < 1$ and $A_t$ is an index for the level of technology. $K_{t-1}$ is the capital purchased and installed in period $t-1$, which becomes productive in period $t$, and $\xi_t$ is a shock to the quality of capital which can be interpreted as obsolesce of the employed capital. Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) use this capital quality shock to simulate the banking crisis preceding the Great Recession in the US. In the context of this analysis, I use a negative capital quality shock to trigger an episode, in which the zero lower bound binds.
At the end of each period the intermediate good producer sells the capital stock that it used for production to the capital producer which repairs the capital, and purchases the capital stock that it is going to use in the next period from the capital producer. To finance the purchase of the new capital at the price $Q_t$ per unit, it issues a claim for each unit of capital it acquires to banks, which trade at the same price. The interest rate the firm has to pay on the loan from the bank is $R_{k,t}$. Under the assumption that the competitive firms make zero profits, the interest rate on their debt will just equal the realized ex-post return on capital. The resale value of the capital used in production depends on the realization of the capital quality shock, and the depreciation rate. Capital evolves according to the following law of motion

$$K_t = (1 - \delta)\xi_t K_{t-1} + I_t. \quad (6)$$

Hence each period the firm in its investment decision maximizes

$$E_t[\beta \Lambda_{t,t+1}(-R_{k,t+1}Q_t K_t + P_{m,t+1}Y_{m,t+1} - W_{t+1}L_{t+1} + (1 - \delta)Q_{t+1}K_{t+1})]$$

with respect to $K_t$. In optimum the ex-post return then is as follows

$$R_{k,t+1} = \frac{P_{m,t+1}\alpha Y_{m,t+1}}{Q_t} + (1 - \delta)Q_{t+1}\xi_{t+1}. \quad (7)$$

Additionally, the optimal choices of labor input yields the first order conditions

$$W_t = P_{mt}(1 - \alpha)\frac{Y_{mt}}{L_t}. \quad (8)$$

**Capital good producers**

The capital good producer’s role in the model is to isolate the investment decision that becomes dynamic through the introduction of convex investment adjustment costs, which is a necessary feature to generate variation in the price of capital. Capital good producers buy the used capital, restore it and produce new capital goods. Since capital producers buy and sell at the same price, the profit they make is determined by the difference between the quantities sold and bought, i.e. investment. Thus they choose the optimal amount of investment to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_{0,t} \left\{ [(Q_t - 1)I_t - f\left(\frac{I_t}{I_{t-1}}\right)I_t] \right\}. \quad (9)$$

The first order condition of the capital producer reads

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}}f'\left(\frac{I_t}{I_{t-1}}\right) - E_t\beta \Lambda_{t,t+1} \left(\frac{I_t}{I_{t-1}}\right)^2 f'\left(\frac{I_t}{I_{t-1}}\right),$$

where the functional form of the investment adjustment costs is

$$f\left(\frac{I_t}{I_{t-1}}\right) = \frac{\eta_t}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2.$$
Retailers

Retailers produce differentiated goods by re-packaging the intermediate goods. They operate under monopolistic competition and face nominal rigidities à la Calvo (1983). As an additional element to smooth the equilibrium dynamics of inflation, it is assumed that in each period the fraction of firms that cannot choose its optimal price, γ, indexes its price to the inflation of the foregoing period. The parameter of price indexation is γ_p.

Aggregate final output, Y_t, is described by a CES aggregator of the individual retailers’ final goods, Y_{ft}

\[ Y_t = \left( \int_0^1 Y_{ft}^{\frac{1}{\epsilon}} df \right)^{-\frac{1}{\epsilon - 1}}. \]

where \( \epsilon > 1 \) is the elasticity of substitution between different varieties of final goods. Thus the demand for its final goods that the retailer faces is

\[ Y_{ft} = \left( \frac{P_{ft}}{P_t} \right)^{-\epsilon} Y_t, \]

where \( P_{ft} \) is the price chosen by retailer \( f \). The aggregate price index is

\[ P_t = \left( \int_0^1 P_{ft}^{1-\epsilon} dt \right)^{-\frac{1}{\epsilon - 1}}, \]

which due to the specific assumptions on the nominal rigidity can be written as

\[ \Pi_t^{1-\epsilon} = (1 - \gamma)(\Pi_t^*)^{1-\epsilon} + \gamma \Pi_{t-1} \gamma_p(1-\epsilon), \]

where \( \Pi_t := \frac{P_t}{P_{t-1}} \), and \( \Pi_t^* := \frac{P_t^*}{P_{t-1}} \). As the retailers’ only input is the intermediate good which is sold by competitive producers, the marginal cost of the retailers equals the price of the intermediate good. Hence, each retailer chooses its optimal price to maximize the sum of its expected discounted profits

\[
E_t \sum_{i=0}^{\infty} (\gamma \beta)^i \Lambda_{t+i} \left\{ \frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} (\Pi_{t+k-1})^{\gamma_p} - \frac{e - 1}{e} P_{m,t+i} \right\} Y_{f,t+i},
\]

subject to the demand constraint. The first order condition for optimal price setting reads

\[
E_t \sum_{i=0}^{\infty} (\gamma \beta)^i \Lambda_{t+i} \left\{ \frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} (\Pi_{t+k-1})^{\gamma_p} - \frac{e - 1}{e} P_{m,t+i} \right\} \left( \frac{P_t^*}{P_{t+i}} \right)^{-\epsilon} Y_{t+i} = 0.
\]

Accordingly, the optimal choice of the price implies

\[ \Pi_t^* = \frac{\epsilon}{\epsilon - 1} \frac{F_t}{Z_t}, \]

where \( F_t \) and \( Z_t \) are defined recursively as

\[ F_t = Y_t P_{mt} + \beta \gamma \Lambda_{t+1} \Pi_t^{\gamma_p} F_{t+1}, \]

\[ Z_t = Y_t + \beta \gamma \Lambda_{t+1} \Pi_{t+1}^{\gamma_p(1-\epsilon)} Z_{t+1}. \]
Equations (13)-(16) constitute the equilibrium conditions, which in a linearized form are equivalent to a New Keynesian Phillips Curve with price indexation. Aggregate output of final goods, \( Y_t \), is related to the aggregate intermediate output, \( Y_{mt} \), in the following way

\[
Y_{mt} = \Delta p_t Y_t, \tag{14}
\]

where \( \Delta_t \) is the dispersion of individual prices, which evolves according to the law of motion

\[
\Delta_{p,t} = \gamma \Delta_{p,t-1} \Pi_t^{\gamma p} + (1 - \gamma) \left( \frac{1 - \gamma \Pi_t^{\gamma p} - \gamma \Pi_t^{\gamma p(c-1)}}{1 - \gamma} \right) \epsilon_t. \tag{15}
\]

The markup \( X_t \) of the monopolistic retailers is the inverse of their marginal costs, which is equivalent to the price of the intermediate good

\[
X_t = \frac{1}{p_{mt}}. \tag{16}
\]

**Banks**

Banks finance their operations by creating deposits, \( D_t \), which are held by households, and by their net worth, \( N_t \). They use their funds to extend loans to intermediate good producers for acquiring capital, \( K_t \), and for the purchases of government bonds, \( B_t \) at their market price \( Q^b_t \). The balance sheet of bank \( j \) is given by

\[
Q_t K_{jt} + Q^b_t B_{jt} = N_{jt} + D_{jt}. \tag{17}
\]

The banks retain the earnings, generated by the return on their assets purchased in the previous period, and add it to their current net worth. Thus, the law of motion for the net worth of a bank is given by

\[
N_{jt} = R_{kt} Q_{t-1} K_{j,t-1} + R_{bt} Q^b_{t-1} B_{j,t-1} - R_{t-1} D_{j,t-1}. \tag{18}
\]

Note that while the interest rate on deposits raised in period \( t-1 \), is determined in the same period, the return of the risky capital assets and risky government bonds purchased in period \( t-1 \) is determined only after the realization of shocks at the beginning of period \( t \). Substituting the balance sheet into the law of motion for net worth yields

\[
N_{jt} = (R_{kt} - R_{t-1}) Q_{t-1} K_{j,t-1} + (R_{bt} - R_{t-1}) Q^b_{t-1} B_{j,t-1} + R_{t-1} N_{j,t-1}. \tag{19}
\]

Bankers continue accumulating their net worth, until they exit the business. Each period, each banker faces a lottery, which determines, regardless of the history of the banker, whether he exits his business or stays in the sector. Bankers exit the business with an exogenous probability \( 1 - \theta \), or continue their operations with probability \( \theta \). The draws of this lottery are i.i.d.. When a banker leaves the sector, it adds his terminal wealth \( V_t \) to the wealth of its household. Therefore, bankers seek to maximize the expected discounted terminal value of their wealth

\[
V_{jt} = \max E_t \sum_{i=0}^{\infty} (1 - \theta)^i \theta^i \beta^{i+1} \Lambda_{t,t+1+i} N_{j,t+1+i} \tag{20}
\]

As banks operate under perfect competition, with perfect capital markets the risk adjusted return on loans and government bonds would equal the return on deposits. However, bankers face an endogenous limit on the amount of funds that households are willing to supply as deposits.
Following Gertler and Karadi (2011), I assume that bankers can divert a fraction of their assets and transfer it to their respective households. However, if they do so, their depositors will choose to withdraw their remaining funds and force the bank into bankruptcy. To avoid this scenario, households will keep their deposits at a bank only as long as the bank's continuation value is higher or equal to the amount that the bank can divert. Formally, the incentive constraint of the bank reads

$$V_{jt} \geq \lambda Q_t K_{jt} + \lambda_b Q^b_t B_{jt},$$

where $\lambda$ is the fraction of loans that the bank can divert, and $\lambda_b$ is the fraction of government bonds it can divert. I calibrate $\lambda_b$ to be smaller than $\lambda$. This is motivated by the fact, that, in general, the collateral value of government bonds is higher than that of loans. The reason is that loans to private firms are less standardized than government bonds contracts. Additionally, information on the credit-worthiness of the government is publicly available, while the credit-worthiness of private firms is often only known to the bank and the firm, and not easy to assess for depositors, making it easier for banks to divert a fraction of their value.

The initial guess for the form of the value function is

$$V_{jt} = \nu_{kj} Q_t K_{jt} + \nu_{bj} Q^b_t B_{jt} + \nu_{nj} N_{jt},$$

where $\nu_{kj}$, $\nu_{bj}$ and $\nu_{nj}$ are time varying coefficients. Maximizing (23) with respect to loans and bonds, subject to (22) yields the following first order conditions for loans, bonds, and $\mu_t$, the Lagrangian multiplier on the incentive constraint

$$\nu_{kj} = \lambda \frac{\mu_{jt}}{1 + \mu_{jt}},$$

$$\nu_{bj} = \lambda_b \frac{\mu_{jt}}{1 + \mu_{jt}},$$

$$\nu_{kj} Q_t K_{jt} + \nu_{bj} Q^b_t B_{jt} + \nu_{nj} N_{jt} = \lambda Q_t K_{jt} + \lambda_b Q^b_t B_{jt}. \quad (24)$$

Given that the incentive constraint binds, a bank's supply of loans can be written as

$$Q_t K_{jt} = \frac{\nu_{bj} - \lambda_b}{\lambda - \nu_{kj}} Q^b_t B_{jt} + \frac{\nu_{nj}}{\lambda - \nu_{kj}} N_{jt}. \quad (25)$$

As (27) shows, the supply of loans decreases with an increase in $\lambda$, which regulates the tightness of the incentive constraint with respect to capital, and increases with an increase in $\lambda_b$, which makes the holding of bonds more costly in terms of a tighter constraint. Plugging the demand for loans into (23), and combining the result with (24) and (25) one can write the terminal value of the banker as a function of its net worth

$$V_{jt} = (1 + \mu_{jt}) \nu_{nj} N_{jt}. \quad (26)$$

A higher continuation value, $V_{jt}$ is associated with a higher shadow value of holding an additional marginal unit of assets, or put differently, with a higher shadow value of marginally relaxing the incentive constraint. Defining the stochastic discount factor of the bank to be

$$\Omega_{j,t} = \Lambda_{t-1,t}((1 - \theta) + \theta (1 + \mu_{jt}) \nu_{nj}),$$

This is in the vein of Meeks et al. (2014), who use the same approach to distinguish between the collateral values of loans and asset backed securities.

The constraint binds in the neighborhood of the steady state. For convenience, I make the assumption that it is binding throughout all experiments.

Detailed derivations are delegated to the appendix.
plugging (28) into the Bellman equation, and using the law of motion for net worth, one can then write the value function as

\[ V_{jt} = E_t [\beta A_{j,t+1} (1 - \theta) N_{j,t+1} + \theta V_{j,t+1}] \]

\[ = E_t [\beta \Omega_{j,t+1} ((R_k,t+1 - R_t) Q_t K_{j,t} + (R_b,t+1 - R_t) Q_t^b B_{j,t} + R_t N_{j,t-1})], \]

and verify the initial guess for the value function as

\[ \nu_{kjt} = \beta E_t \Omega_{j,t+1} (R_k,t+1 - R_t), \quad \text{(28)} \]

\[ \nu_{bjt} = \beta E_t \Omega_{j,t+1} (R_b,t+1 - R_t), \quad \text{(29)} \]

\[ \nu_{njt} = \beta E_t \Omega_{j,t+1} R_t. \quad \text{(30)} \]

**Aggregation of financial variables**

To facilitate aggregation of financial variables, I assume that banks share the same structure to the extent that they derive the same respective values from holding loans and bonds, and from raising deposits (i.e., \( \forall j: \nu_{kjt} = \nu_{kt}, \nu_{bjt} = \nu_{bt}, \nu_{njt} = \nu_{nt} \)). Furthermore, I assume that all banks have the same ratio of capital assets to government bonds, \( \zeta_t \equiv \frac{Q_t K_t}{Q_t^b B_t} \), in their portfolio. As an implication, the leverage ratio of banks does not depend on the conditions that are specific to individual institutes, and all banks share the same weighted leverage ratio \(^9\)

\[ \phi_t \equiv \frac{\nu_{nt}(1 + \zeta_t)}{(\lambda - \nu_{kt})(1 + \frac{1}{\lambda} \zeta_t)} = \frac{Q_t K_t + Q_t^b B_t}{N_t}. \quad \text{(31)} \]

Note that the lower divertability of government bonds relative to capital assets, allows the bank to increase its leverage ratio, compared to a scenario in which banks only hold capital assets. The aggregate balance sheet constraint reads

\[ Q_t K_t + Q_t^b B_t = D_t + N_t. \quad \text{(32)} \]

The net worth of the fraction of bankers that survive period \( t - 1 \) and continue operating in the banking sector, \( \theta \), can be written as

\[ N_{ot} = \theta \left[ R_{kt} Q_{t-1} K_{t-1} + R_{bt} Q_{t-1}^b B_{t-1} - R_{t-1} D_{t-1} \right]. \quad \text{(33)} \]

A fraction \( (1 - \theta) \) of bankers leaves the business. There is a continuum of bankers, and the draws out of the lottery, which determines whether a banker stays in business or exits the sector, are iid. Hence, by the law of large numbers, it follows that the share of assets that leaves the sector is a fraction \( (1 - \theta) \) of the total assets. At the same time, new bankers enter the sector. New bankers are endowed with "start-up funding" by their households. The initial endowment of the new bankers is proportionate to the assets that leave the sector. The net worth of the new bankers, \( N_{nt} \), can be written as

\[ N_{nt} = \omega \left[ Q_{t-1} K_{t-1} + Q_{t-1}^b B_{t-1} \right], \quad \text{(34)} \]

where \( \omega \) is calibrated to ensure that the size of the banking sector is independent of the turnover of bankers. Aggregate net worth, \( N_t \), is then the sum of the net worth of old and new bankers

\[ N_t = N_{ot} + N_{nt}. \quad \text{(35)} \]

\(^9\)Details are delegated to the appendix.

\(^{10}\)Note that if the collateral values of capital assets and bonds were the same \( (\lambda = \lambda_b) \), the leverage ratio would take the same form as in Gertler and Karadi (2011), or in Kirchner and van Wijnbergen (2016)
Fiscal policy

The fiscal sector closely follows the structure in van der Kwaak and van Wijnbergen (2013) and van der Kwaak and van Wijnbergen (2015). The government finances its expenditures, $G_t$, by issuing government bonds, which are bought by banks, and by raising lump sum taxes, $T_t$. Government spending is exogenous and follows an AR(1) process

$$G_t = G_0 g_t, \quad (36)$$

and

$$g_t = \rho_g g_{t-1} + \epsilon^g_t, \quad (37)$$

where $G$ is the steady state government consumption, $\rho_g$ is the autocorrelation of government consumption, and $\epsilon^g_t$ is a shock to government spending. Taxes follow a simple feedback rule, such that they are sensitive to the level of debt and to changes in government expenditures

$$T_t = T + \kappa_b (B_{t-1} - B) + \kappa_g (G_t - G), \quad (38)$$

where $T$ and $B$ are the steady state levels of tax revenue and government debt, respectively. $\kappa_b$ is set to ensure that the real value of debt grows a rate smaller than the gross real rate on government debt. As shown by Bohn (1998), this rule is a sufficient condition to guarantee the solvency of the government. If $\kappa_g$ is set to zero, increases in government expenditures are entirely debt-financed. In turn, when $\kappa_g = 1$, changes in government spending are tracked one-to-one by changes in taxes.

To allow for the calibration of a realistic average maturity of government debt, bonds are modeled as consols with geometrically decaying coupon payments, as in Woodford (1998) and Woodford (2001). A bond issued in period $t$ at the price of $Q^b_t$, pays out a coupon of $r_c$ in period $t + 1$, a coupon of $\rho_c r_c$ in period $t + 2$, a coupon of $\rho^2 c r_c$ in $t + 2$, and so on. Setting the decay factor $\rho_c$ equal to zero captures the case of a one-period bond in which the entire payoff of the bond is due in period $t + 1$. Setting $\rho_c = 1$ delivers the case of a perpetual bond. The average maturity of a bond of this type is $1/(1 - \beta \rho_c)$. For investors, this payoff structure is equivalent to receiving the coupon $r_c$ and a fraction, $\rho_c$, of a similarly structured bond in period $t + 1$. The beginning-of-period debt of the government can thus be summarized as $(r_c + \rho_c Q^b_t) B_{t-1}$.

At the beginning of each period, the government has the option to default and write off a fraction of its debt, $D \in (0, 1)$. Investors take this into account, and demand a higher return on government bonds, when the expected probability of a sovereign default, $\Delta^d_t$, increases. The return to government bonds, adjusted for default risk, can thus be written as

$$R_{b,t} = (1 - \Delta^d_t * D) \left[ \frac{r_c + \rho_c Q^b_t}{Q^b_{t-1}} \right]. \quad (39)$$

The flow budget constraint of the government reads

$$G_t + R_{b,t} Q^b_{t-1} B_{t-1} = Q^b_t B_t + T_t, \quad (40)$$

or:

$$G_t + (1 - \Delta^d_t * D) \left[ \frac{r_c + \rho_c Q^b_t}{Q^b_{t-1}} \right] Q^b_{t-1} B_{t-1} = Q^b_t B_t + T_t. \quad (40)$$

Linking the probability of a sovereign default to the level of public debt or the debt-to-GDP ratio is common in the literature (see, e.g., Eaton and Gersovitz (1981), Arellano (2008) or Leeper and Walker (2011)). A higher level of public debt implies a higher debt service, and, in turn, requires higher tax revenues to service the interest rate payments. As tax increases are not popular and only up to a maximum level politically feasible, it is plausible to posit a maximum capacity of
levying taxes, or fiscal limit. With an increasing public debt, the economy moves closer to the fiscal limit.

The probability of a sovereign default is described by the logistical distribution function

$$
\Delta_t^d = \frac{\exp\left(\eta_1 + \eta_2 \frac{B_t}{4Y_t}\right)}{1 + \exp\left(\eta_1 + \eta_2 \frac{B_t}{4Y_t}\right)},
$$

which depends on the debt-to-GDP ratio, and is depicted in figure (2). The fiscal limit function is pinned down by the parameters $\eta_1$ and $\eta_2$. I use the results of the structural estimation of an RBC model on Italian data by Bi and Traum (2012a) to calibrate these parameters.

**Monetary policy and good market clearing**

The policy tool of the central bank in this economy is the nominal interest rate, $i_t$. Monetary policy follows a Taylor rule. The nominal interest rate is non-negative. When the ZLB constraint is not binding, the central bank reacts to fluctuations in the net inflation rate, $\pi$, and of the log deviation of the real marginal cost from the viewpoint of the final good producer, $\hat{P}_{mt}$

$$
i_t = \max(0, [i + \kappa_\pi \pi_t + \kappa_y \hat{P}_{mt}]).
$$

For the model to render a unique rational expectation equilibrium in the neighborhood of the deterministic steady state, it has to hold that $\kappa_\pi > 1$. The Fisher equation is $1 + i_t = R_t E_t \Pi_{t+1}$. Closing the good market clearing condition reads

$$
Y_t = C_t + I_t + f\left(\frac{I_t}{I_{t-1}}\right) I_t + G_t.
$$
The stochastic disturbances that drive the dynamics of the model are the capital quality shock, and the government spending shock, which follow the AR(1)-processes

\[ g_t = \rho_g g_{t-1} + \epsilon_g^t, \quad (44) \]
\[ \xi_t = \rho \xi_{t-1} + \epsilon^\xi_t. \quad (45) \]

3. Calibration and solution method

Calibration

The calibration of the model is motivated by the case of the Italian economy, which represents a case of a large, relatively closed economy with high public debt, and recurring periods of high interest rate spreads in the last decades. The model is calibrated to quarterly frequency. Table 1 lists the parameter values used in the model. A major source for the parameter values is Bi and Traum (2012a). Bi and Traum (2012a) estimate an RBC model with a sovereign default indicator on Italian data from 1999.Q1 and 2010.Q3.

Parameter \( \beta \), the discount factor of the household, is set to 0.995, instead of the conventional 0.99. Increasing the discount factor implies a lower interest rate at the steady state of the model moving it closer to the ZLB. As a consequence, the solution method, which is discussed in the next subsection, reliably generates solutions for scenarios, in which the economy remains up to five periods at the ZLB, and across different calibrations for the debt-to-GDP ratio. The coefficient of relative risk aversion is set to 1. In addition, I follow (Bi and Traum, 2012a) and choose for the deterministic steady state a debt-to-GDP ratio of 1.19, and an output share of government spending of 0.1966 to match the respective means in their sample. In accordance with their estimation results, I set the parameter for consumption habit, \( h \), to 0.14, the persistence and standard deviation of the technology shock, \( \rho_a \) and \( \sigma_a \) to 0.96 and 0.01, and the persistence and standard deviation of the government spending shock, \( \rho_g \) and \( \sigma_g \) to 0.84 and 0.01, respectively. Furthermore they obtain the values 0.3 for \( \kappa_b \) and 0.53 for \( \kappa_g \). Under the assumption of a haircut of 37.8 percent on the outstanding debt in case of a sovereign default, their estimation results imply a default function with the parameter values \( \eta_1 = -21.5285 \) and \( \eta_2 = 3.4015 \).\(^{11}\)

The values of the parameters of the default function reflect that the estimation by Bi and Traum (2012a) is based on a sample that largely contains years in which the market for Italian government bond was calm. They imply a quarterly default rate of 0.0048 on sovereign bonds, and a low sensitivity of the default rate to movements in the debt-to-GDP ratio at the deterministic steady state. Similarly, the calibration of the parameters associated with the government spending shock and the technology shock reflects a mixture of calm years and crisis years. I choose these parameter values by Bi and Traum (2012a) as my benchmark calibration, since an empirically plausible calibration of the default function is crucial for the assessment of the role of fiscal stress for the multiplier.

In the calibration of the parameters associated with the different firm sectors, I borrow from the model by Gertler and Karadi (2011), which is the fundament of the framework at hand.\(^{12}\) The effective capital share, \( \alpha \) is 0.33. The depreciation rate is 0.025 in the deterministic steady state. Parameter \( \eta_i \), which governs the investment adjustment costs is set to 1.72 and the elasticity of intra-temporal substitution, \( \eta \), is set to 4.167. The Calvo parameter, \( \gamma = 0.779 \) implies an average price duration of roughly four and a half quarters. The degree of price indexation, \( \gamma_p \) is rather low.

\(^{11}\) For more details on the estimation procedure, see Bi and Traum (2012a).

\(^{12}\) In turn, Gertler and Karadi (2011) borrow most of their parameter values from Primiceri, Schaumburg, and Tambalotti (2006), who estimate a medium scale model on US data.
Further parameter values that I use from Gertler and Karadi (2011), are the inverse of the Frisch elasticity, $\varphi$, the feedback parameters in the Taylor rule, $\kappa_\pi$ and $\kappa_y$, the persistence of the capital quality shock $\rho_\xi$, and some parameters of the banking sector. In the calibration of the banking sector, I follow a similar strategy as Gertler and Karadi (2011), and choose the fractions of the assets that the banks can diverge, $\lambda$ and $\lambda_b$, the survival probability of the bankers, $\theta$, and the transfer to new bankers, $\omega$, in order to target a steady state leverage ratio of 4, an average time horizon of the bankers of a decade, and the steady state spreads of the returns on the banks assets over the deposit rate. For the steady state spread of the return on capital over deposits I use the same target as in Gertler and Karadi (2011). For the steady state spread of government bonds over deposits I take the estimate by Bocola (2016) as a guideline.

The difference of the steady state spreads of the two assets is reflected by the values of the respective divertability parameters, $\lambda$ and $\lambda_b$. In the deterministic steady state these two parameters are linked by the relation

$$\frac{\lambda}{\lambda_b} = \frac{R_k - R}{R_b - R}.$$  

The coupon rate on the long-term government bond, $r_c$, is set to 0.04, and the rate of decay of the bonds, $\rho_c$, is set to 0.96 as in van der Kwaak and van Wijnbergen (2015). The standard deviations of the monetary policy shock and the capital quality shock are set to 0.01. The parameter $\chi$, which weighs the disutility of labor is chosen such as to balance the labor supply equation in the deterministic steady state.

### Solution method

To deal with the occasionally binding ZLB constraint, I employ the piecewise-linear approach by Guerrieri and Iacoviello (2015). This approach treats models with occasionally binding constraints as models with two different regimes: one in which the constraint is binding and one in which it is slack. The model is linearized around a reference point, here the deterministic steady state. Guerrieri and Iacoviello (2015) refer to this state as the reference regime, and other one the alternative regime. Their approach requires two conditions to be fulfilled. First, the conditions for existence of a rational expectation equilibrium laid out by Blanchard and Kahn (1980) have to hold in the reference regime. Secondly, if shocks move the economy away from the reference and into the alternative regime, it has to return to the reference regime in finite time. Both conditions hold in the case at hand.

In this paper, the ZLB constraint is slack in the reference regime, and binding in the alternative regime. At the reference regimes, the rational expectation equilibrium of the model can be described as

$$\mathcal{A} E_t X_{t+1} + \mathcal{B} X_t + \mathcal{C} X_{t-1} + \mathcal{E} \epsilon_t = 0, \quad (46)$$

where $X_t$ is the vector of $n$ endogenous variables, $\epsilon_t$ is the vector of $m$ exogenous disturbances, and the elements of the matrices $\mathcal{A}, \mathcal{B}, \mathcal{C}$, (each of size $n \times n$) and $\mathcal{E}$ (of size $n \times m$) are functions of the structural parameters of the model. At the alternative regime, the rational expectation equilibrium of the model can be described as

$$\mathcal{A}^* E_t X_{t+1} + \mathcal{B}^* X_t + \mathcal{C}^* X_{t-1} + \mathcal{D}^* + \mathcal{E}^* \epsilon_t = 0. \quad (47)$$

$\mathcal{A}^*, \mathcal{B}^*, \mathcal{C}^*, \text{ and } \mathcal{E}^*$ are matrices that are analogous to $\mathcal{A}, \mathcal{B}, \mathcal{C}$, and $\mathcal{E}$. $\mathcal{D}^*$ is a vector of parameters of size $n$, which corrects for the fact that the linearization is carried out around the deterministic steady state, in which the reference regime applies. Given initial conditions $X_0$ and
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
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<tbody>
<tr>
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<td>η_2</td>
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<td>ρ_a</td>
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<tr>
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<td>std. of i-shock</td>
<td>0.01</td>
<td></td>
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<tr>
<td>σ_ξ</td>
<td>std. of ξ-shock</td>
<td>0.01</td>
<td></td>
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<tr>
<td>σ_g</td>
<td>std. of g-shock</td>
<td>0.01</td>
<td>Bi and Traum (2012)</td>
</tr>
<tr>
<td>σ_a</td>
<td>std. of a-shock</td>
<td>0.01</td>
<td>Bi and Traum (2012)</td>
</tr>
</tbody>
</table>
a sequence of shocks, the solution to this model is a policy function
\begin{equation}
X_t = P_t X_{t-1} + R_t + D_t \epsilon_t,
\end{equation}
where matrices $P_t$ ($n \times n$), $R_t$ ($n \times 1$), and $D_t$ ($n \times m$) are regime-dependent and $R_t = 0$ at the reference regime. The solution algorithm starts with a guess for the period, in which the economy returns to the reference regime, and for the regime in each period up to the final return period. Then it solves for the policy function at the date in which the economy has just returned to the reference regime, plugs the policy function into the equilibrium conditions for the alternative regime, and applies backward induction to trace the equilibrium path back to the initial period, applying either system (46) or system (47) at any given period along the path, depending on whether the economy is at the ZLB or not according to the guess. Finally, the initial guess is either verified or updated until a guess has been verified.

How long the economy remains at the ZLB depends on the model structure and on the size of the shock that pushes the economy to the ZLB. An attractive property of the solution is that expectations of how long the ZLB regime will last, already affect the decisions by the agents contemporaneously. An advantage of using the piecewise-linear approach is that, at it relies on perturbation methods, it is easily applicable to medium-scale models such as the one being used for the analysis in this paper.

### 4. Crisis scenario and government spending stimulus

To trigger the crisis that moves the economy to the ZLB, I simulate a negative shock, as it is done in Gertler and Karadi (2011). I consider different lengths of the duration of the ZLB and compare government spending multipliers across different initial calibration of the debt-to-GDP ratios and different durations of the ZLB period. For each steady state value of the debt-to-GDP ratio, and for each desired duration of the ZLB, I adjust the size of the capital quality shock accordingly.

In order to illustrate the effects of the fiscal stress channel for the output effects of the shock, I compare the full model described in section 2 with a model in which the fiscal stress channel is shut off. Here, I eliminate equation (41) from the model, and set variable $\Phi_t$ to a constant value (i.e., $\Phi_t = \Phi \ \forall t$). In all other aspects the two models are identical. I refer to the latter model as the model without fiscal stress.

#### 4.1. Crisis scenario

Figure (3) illustrates the consequences of a fall in capital quality of four percent that pushes the economy to the ZLB for four quarters. In this particular case, the debt-to-GDP ratio is calibrated to 1.19, which is the average debt-to-GDP ratio in the sample considered by Bi and Traum (2012a) in the estimation of their structural model. The dashed lines represent the responses of the variables in the linear case, which ignores the ZLB and the solid lines depict the responses of the

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\[13\] In their definition of a solution for the model, Guerrieri and Iacoviello (2015) focus at the case, in which there is only a shock in period 1. However, their software package Occbin allows for shocks in subsequent periods as well.

\[14\] This stands in contrast to algorithms, in which the model’s return to the reference regime, is determined each period anew, according to a stochastic process (see, e.g., Eggertson and Woodford (2003)).

\[15\] Gertler and Karadi (2011) liken this shock to a sudden economic depreciation or obsolescence of capital. Often, to trigger a zero bound episode, a shock to the discount factor that raises the agents desire to save is used. However, in the model at hand this shock has the undesirable feature that it increases aggregate investment in crisis times. The capital quality shock circumvents this pitfall.

\[16\] The value of $\Phi_t$ in the deterministic steady state, is the same in both models.
variables generated by the piecewise linear solution, which takes the ZLB into account. Black lines show the responses of the full model. Blue lines show the model in which the fiscal stress channel is eliminated.

In each scenario in figure (3), the exogenous decline in capital quality effectively reduces the productive capital stock on impact. As a consequence, aggregate output falls and so does consumption of households. The lower stock of effective capital reduces the marginal product of labor, and consequently the demand for labor falls, leading to a decline in equilibrium hours worked. An additional consequence of the lower quality of capital is that the price of capital assets falls. With the subsequent deterioration of the banks’ balance sheets, their shrinking net worth forces banks to sell off assets, amplifying the fall in the price of capital. The leverage ratio increases, and so does the spread of the return on capital (loans of the bank to firms) over the return on deposits. The ensuing credit crunch causes aggregate investment to drop. A second reaction by banks to the capital quality shock is to change the composition of their asset holdings. In a flight to the relatively less affected assets, they increase the share of government bonds in their portfolio. Thus, in contrast to the fall in capital assets, the amount of government bonds held by banks increases.

How does the fiscal stress affect the dynamic consequences of the capital quality shock? As the sovereign default probability goes up together with the debt-to-GDP ratio, the price of government bonds falls by more than in the case, in which the fiscal stress channel is inactive. Similarly, the spread on bonds over deposits increases more sharply. With the price of bonds being more sensitive to the shock, bonds lose some attractiveness as a safe haven and the portfolio shift towards government bonds is less pronounced. Instead, the banks net worth falls more than in the scenario, in which the fiscal stress channel is inactive, and the leverage ratio as well as the credit spreads increases by more in this case. Hence, the contraction in the supply of credit is stronger than in the case without the fiscal stress channel, deepening the fall in aggregate investment and output. The pro-cyclical amplification of the financial accelerator is therefore stronger, when the fiscal stress channel is active.

What is the role of the ZLB constraint for the equilibrium dynamics after the negative capital quality shock? In response to the shock, the central bank lowers its nominal rate to counter the deflationary push and to stimulate the economy. In the simulation in which the ZLB is neglected, the nominal policy rate is lowered to minus 2 percentage points. In the simulation, in which the ZLB is accounted for, the nominal rate falls to zero and the ZLB binds for 4 periods (solid lines). When the ZLB is binding, the ability of the central bank to stimulate aggregate demand and to offset the downturn is constrained. Thus, while the dynamic consequences of the shock are qualitatively similar to the simulation, in which the ZLB is neglected, the size of the recessionary impact of the shock is larger. At its trough, output decreases in the case with the ZLB and the fiscal stress channel by roughly 4,5 percent compared to 4 percent in the case without the ZLB constraint. With the magnified contraction of real activity, the distress in the banking sector increases, amplifying the rise in the spreads and the leverage ratios and the decline in asset prices, net worth and aggregate investment. As in the scenario without the ZLB, the fiscal stress channel adds to the vulnerability of the banks’ balance sheets, and magnifies the financial accelerator in the downturn.
Figure 3: Dynamic consequence of a negative capital quality shock. The size of the shock is calibrated to make the zero lower bound binding for 4 periods. The debt-to-GDP ratio is calibrated to 1.19.
4.2. The government spending shock

The case, when the ZLB is not binding

Against the backdrop of the crisis described in the foregoing subsection, I simulate a government spending stimulus of the size of one percent of steady state output (i.e. five percent of steady state government expenditures) and compare the effects across the four scenarios introduced in the last subsection. Let me first focus on the analysis of the impulse responses to a government spending shock in the case in which the zero lower bound does not bind. In figure (4) the respective scenarios are depicted as the dashed lines (blue - without the fiscal stress channel, black - with an active fiscal stress channel).\textsuperscript{17} As figure (4) illustrates the contribution of fiscal stress is negligible, when the ZLB is not binding. However, before I turn to the ZLB case, I briefly discuss how the features embedded in the model affect the dynamic consequences of the government spending shock.

As the model contains several features, which affect the transmission of the shock, I start with a brief description of those effects that are independent of financial frictions or fiscal stress. The government spending shock increases government debt and stimulates output. Private activity is crowded out, however. Since households are Ricardian, and preferences are additive separable in consumption and leisure, the increase in government spending induces a wealth effect on the labor supply.\textsuperscript{18} As in many models, consumption falls and the labor supply increases after the shock, leading to increasing equilibrium labor hours and a decline in the real wage. With the higher labor input the marginal product of labor decreases. In the presence of sticky prices, the average real marginal cost increase and the average markup declines with the increase in production. As a result of the falling markup, the demand for labor increases for any given real wage, contributing to the increase in equilibrium labor hours and output.\textsuperscript{19}

As highlighted by Monacelli and Perotti (2008), the increase in labor hours raises the marginal product of capital. In equilibrium this is accompanied by an increase in the loan rate. This has the effect that investment becomes less attractive, leading to a decline in the capital stock as well as in the price of capital. The firms’ demand for loans from banks contracts. The resulting decline of investment in response to the increase in government spending, that is essential for the link between fiscal stress and the multiplier in my model, is consistent with empirical evidence. For the US, a decline in investment in response to the increase in government spending, is observed in Blanchard and Perotti (2002), Mountford and Uhlig (2009), Ramey (2011). Evidence from cross-country panels provided by Corsetti et al. (2012) and by Ilzetzki et al. (2013) similarly points to a decline in investment in response to an increase in government spending.

As Kirchner and van Wijnbergen (2016) show, the crowding-out of investment is amplified through a contraction in the supply of loans, if government bonds are held by leverage constrained as it is the case in this model. Banks reduce the supply of loans for two reasons: First, as they incur capital losses due to the decline in capital prices, their net worth shrinks, tightening their leverage constraints. Additionally, the price of government bonds falls together with the price of capital, due to arbitrage on financial markets, further tightening the leverage constraint.

\textsuperscript{17}For each of the scenarios, I simulate the equilibrium dynamics once in response to the capital quality shock and the government spending shock, and once with only the capital quality shock. Then, I subtract the results of the latter simulation from the former with both shocks. Figure (4) displays the results: the isolated effect of the government spending shock on the equilibrium path.

\textsuperscript{18}For a discussion of the wealth effect on the labor supply, see, e.g., Christiano and Eichenbaum (1992) or Baxter and King (1993).

\textsuperscript{19}For an analysis of the role of markup shifts for the transmission of government spending shocks in the New Keynesian model, see: Monacelli and Perotti (2008).
The fall in the prices of capital assets and bonds drive up the spread of the return on bonds over the return on deposits. Lastly, the reduction of the supply of loans increases expected future returns on loans. Hence, the spread of the return on capital assets (loans from the banks to the firms), over the return of deposits increases. A second reason reason for the contraction of the loan supply is that banks change the composition of their asset portfolio. The increase in public debt increases their holdings of government bonds. Given the constraint on the size of banks’ balance sheets, the portfolio shift towards government bonds exerts further pressure on the supply of loans to firms. Overall, the financial accelerator mechanism that is implied by constrained banks amplifies the crowding out of investment compared to a situation with unconstrained investors in capital assets, and lowers the government spending multiplier.

Now let me turn to the effect of fiscal stress on the government spending multiplier. Most of the literature on the interaction of fiscal multipliers and fiscal stress, theoretical as well as empirical, focusses on the possibility that higher fiscal stress decreases the impact of government spending shocks. Theoretical studies that focus on the effect of fiscal retrenchment suggest that, for instance, an improvement of the expectations of future growth (Bertola and Drazen (1993)), or a decrease in risk premiums on government bonds, enhanced financial intermediation and increasing investment (e.g. Alesina and Perotti (1997), Corsetti et al. (2013) or van der Kwaak and van Wijnbergen (2013)) counteract the otherwise detrimental effects of fiscal retrenchment on output growth. I follow the authors who focus in their argument on the channel of financial intermediation and changes in risk premiums.

The response to the government spending shock in the linearized model with fiscal stress depicted as dashed black curves in figure (4). As one can see, the contribution of the fiscal stress channel to the response of the financial and real variables to the government spending shock is very small. The main difference to the scenario without fiscal stress is that now the default probability responds to the movements in the debt-to-GDP ratio. As the initial increase in GDP is larger in percentage terms than the initial increase in public debt, the debt-to-GDP ratio decreases on impact. This initial fall in the default probability after a fiscal expansion is in line with empirical findings by Born, Müller, and Pfeifer (2015) and Strobel (2016) who find that in periods of a higher fiscal stress, bond yield spreads increase on impact after contractionary fiscal shocks. Already after the second quarter, the default probability in the model rises above the steady state, since the increase in debt is more persistent than the output response. Through equation (41) the variation in the default probability contributes to the fall in price of bonds and the increase in the return on bonds, and as prices and returns of assets move together, it contributes to the fall in the price of capital and the increase in the return on capital as well. The contraction of investment and capital are slightly stronger, and the response of output slightly weaker than in the model without fiscal stress. The size of this effect is, however, very small.

The case, when the ZLB is binding

Now let me turn to the case, in which the ZLB becomes binding. At the ZLB, the effects of government spending shocks change drastically. As in Christiano et al. (2011) and Eggertson (2011), the output stimulus of the government spending shock is far stronger at the ZLB. The capital quality shock that triggers the ZLB period, leads to a deep recession associated with strong deflationary tendencies. When the nominal interest rate is constrained to be zero, expected deflation translates into a rise in the real interest rate, which incentivizes households to delay consumption and which makes investment less attractive to firms. In this situation, the increase
in government spending dampens the fall in aggregate demand and counters the deflationary tendencies. As figure (4) shows, the positive impact of the government shock on inflation is far stronger at the ZLB. As long as nominal rate is at zero, the push in inflation dampens the increase in the real interest rate. As a consequence, it becomes more attractive for firms to invest and aggregate investment is now even crowded-in by the government spending shock. The surge in investment demand results in a growing capital stock and an increasing price of capital assets at the ZLB. Hence, at the ZLB the effect of government spending is to bolster the net worth of banks, to lower their leverage ratio and to expand their ability to hold assets and to supply loans. This further amplifies the investment boom, and contributes to the stronger rise in aggregate output after the government spending shock. The consumption of Ricardian households is still crowded out by the government spending shock, due to the wealth effect on the labor supply, but this effect is much weaker than away from the ZLB.

Similarly, the role of the fiscal stress channel for the effects of the government spending shock is reversed at the ZLB. Here, the expansion of output is strong enough to decrease the debt-to-GDP ratio, and thus the default probability of government bonds. Consequently, the price of bond increases by more than in the simulation without the fiscal stress channel. In equilibrium, this positive effect spills over to the price of capital. As a result, the net worth of banks is further bolstered, and the leverage ratio is lower than in the scenario with fiscal stress. Whereas the spreads of credit and government bonds over the deposit rate still increase slightly, when the fiscal stress channel is inactive, they fall in the scenario, in which the fiscal stress channel further contributes to a decline in the return on capital and bonds as well as to falling spreads. The crowding-in of investment is more pronounced and the capital accumulation accelerated. Overall, the fall of the sovereign default probability due to the strong expansion of output after the government spending stimulus further raises the government spending multiplier.

Figure (5) displays the cumulative government spending multiplier over a time horizon of 20 quarters for different steady state levels of public indebtedness and different lengths of the ZLB episode. Since its accession to the Eurozone, the stock of Italian public debt fluctuated between 100 and 132 percent of Italy's GDP. The debt-to-GDP ratios in the four panels are chosen accordingly. For each of these calibrations, I conduct simulations with ZLB episodes up to five periods. As one can see, in the linear scenario the fiscal stress channel hardly affects the government spending multiplier. The blue and the black dashed lines are virtually indistinguishable.

For the simulations, in which the ZLB constraint is accounted for, three results stand out. First, at the ZLB government spending multipliers can become much larger than in normal time, regardless of the level of public debt. Secondly, across different levels of public indebtedness the government spending multiplier increases quickly with the duration of the ZLB period. Longer ZLB periods reflect deeper recessions, which are triggered by larger negative capital quality shocks. The deeper the crisis, the stronger is the deflationary pressure induced by the shock. Consequently, an expansion of government spending dampens a stronger deflation for more periods, until the economy leaves the ZLB. Thus, the spending stimulus lowers the real interest rate for longer and stimulates investment by more, the longer the ZLB period. In these simulations, the government spending multiplier quickly becomes larger than one and reaches up to roughly 2.3. Again, this result

\[20\] In a larger sensitivity analysis, I tested debt-to-GDP ratios between 0.6, the threshold requested in the Maastricht treaty, and 1.3. The fiscal stress channel is quantitatively negligible for small values of the debt-to-GDP ratio. The results are qualitatively robust to an extension of the parameter space and are available upon request.
Figure 4: Dynamic consequence of a government spending shock of one percent of steady state output. The size of the shock is calibrated to make the zero lower bound binding for 4 periods. The debt-to-GDP ratio is calibrated to 1.19.
Thirdly, the positive contribution of the fiscal stress channel to the multiplier is larger for longer ZLB episodes. This result follows naturally as a stronger expansion of aggregate output at longer ZLB periods, leads to a stronger decline in the debt-to-GDP ratio and hence as well the default probability in the model. This improves the financial conditions of banks, and facilitates a larger credit and investment volume.

Also, as is shown in figure (6), the contribution of the fiscal stress channel is generally larger, the higher the debt-to-GDP ratio.\(^{21}\) This is due to the higher sensitivity of the default probability to fluctuations in the debt-to-GDP ratio for higher levels of public debt as implied by equation (41).\(^{22}\) For the case without fiscal stress, the relationship between the debt-to-GDP ratio and the multiplier is generally negative at the ZLB. A higher debt-to-GDP ratio implies a larger share of government bonds in the portfolio of banks, serving as a buffer from the consequences of the recessionary capital quality shock, and dampening the amplification of the crisis by the financial accelerator. Thus, the recession and the deflation triggered by the crisis shock are less severe for higher levels of public debt. The inflationary impulse by the government spending stimulus counters a less severe deflation and therefore affects the real interest rate by less. Accordingly, when the fiscal stress channel is inactive, the stimulus of government spending to investment and

\(^{21}\)The panel for a duration of the ZLB of 5 periods is omitted. The size of the capital shock needed to obtain a duration of 5 periods in this model varies irregularly with the calibration of the debt-to-GDP ratio. As the size of the initial crisis shock is itself a determinant of the multiplier, the results are therefore strongly distorted.

\(^{22}\)The non-linear shape of this function is taken from Bi and Traum (2012a) and is in line with results by panel data studies on the drivers of sovereign yield spreads in Europe, which find a stronger sensitivity of the spreads to fundamentals in crisis times (see, e.g., Manganelli and Wolswijk (2009), Beirne and Fratzscher (2013))
Figure 6: Cumulative Government Spending Multipliers over a time horizon of 20 quarters
output is weaker, for higher levels of public debt. The same rationale holds for the cases, in which the fiscal stress channel is active, but weak due to low levels of the steady state debt-to-GDP ratio.

When the fiscal stress channels is active and the debt-to-GDP ratio is rather high, the default probability is more sensitive to the fall in the debt-to-GDP ratio. Therefore, in this case fiscal stress amplifies the crisis and implies a deeper fall in output and the inflation rate. This makes the impact of the government spending shock again stronger on the real interest rate. Additionally, an active fiscal stress channel implies, a sharper fall in the default probability after the government spending shock, which itself increases the output stimulus and the multiplier.

The result that the government spending multiplier at the ZLB, can be larger when a country is higher in debt, is in contrast with the finding by Corsetti et al. (2013), who argue in the context of a small New Keynesian model with banks à la Cúrdia and Woodford (2011) that fiscal stress affects the multiplier negatively at the ZLB. However, it is in line with the results by Aloui and Eyquem (2016), who find that extending the model by Corsetti et al. (2013) with capital accumulation and distortionary taxes overturns their result and establishes a positive relationship between the debt-to-GDP ratio and the government spending multiplier at the ZLB. As my analysis differs from theirs not only in the model features, but also in the solution method, a close comparison of the results is, however, not straightforward.

5. Conclusion

In the sovereign debt crisis in the Eurozone, several countries have conducted government spending cuts against the backdrop of an economic environment, which featured a high degree of fiscal stress and a monetary policy that was constrained by the ZLB on nominal interest rates. Both features have been found to be potentially important determinants of the size of the government spending multiplier. This paper investigates on the size of the government spending multiplier in a model that takes account of both features. I find that the presence of fiscal stress does not undo the widespread notion that multipliers are large at the ZLB. In fact, the presence of a fiscal stress channel can even increase the multiplier at the ZLB.

To the extent that shocks to government spending change the debt-to-GDP ratio, they affect the degree of fiscal stress, and thus the prices of government bonds and capital assets. As at the ZLB an expansion of government spending prevents a deflationary trap, it raises inflation expectations and lowers the real interest rate. Thereby, it stimulates investment and leads to a large output response. The longer the crisis episode in which the ZLB holds, the stronger the expansionary effect of a positive government spending shock.

Since the increase in aggregate output is strong enough to lower the debt-to-GDP ratio despite the increasing amount of outstanding government bonds. A fiscal expansion lowers the degree of fiscal stress, takes pressure of bank's balance sheets, improves their ability to lend and contributes to the investment boom. This holds particularly when the government is highly indebted, and the default probability reaction to variations in the debt-to-GDP ratio is more sensitive. Taking into account variations in fiscal stress, actually increases the government spending multiplier.

With my analysis, I operate at the intersection of two distinct strands of the literature, which either focus on the effects of fiscal stress or the effects of the ZLB on the government spending multiplier. My result is in line with the gist of the theoretical literature on multipliers at the ZLB, which predominantly finds large multipliers. My analysis confirms that this finding is robust to the presence of high government debt and a fiscal stress channel in a medium scale model.
As a caveat, the dependence of the solution method on linear equilibrium conditions, implies that potentially interesting effects of uncertainty are unaccounted for. Thus adjusting the model to make it suitable for a solution method that allows to capture effects of non-linearities is a interesting path for future research. Alternatively, for a more complete account of a sovereign debt crisis, the analysis can be extended to include other important aspects that can determine the size of government spending multiplier, and that are omitted from this analysis, such as open economy considerations, the role of private debt, unconventional monetary policy and distortionary taxation. The investigation of the effects of fiscal policy remains a fruitful field for future research.

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